Pairing and shell effects in the transport coefficients of collective motion *

F.A.Ivanyuk[1,2] and H.Hofmann[2]

1) Institute for Nuclear Research, Kiev-28, Ukraine, e-mail: ivanyuk@kinr.kiev.ua

2) Physik-Department der TUM, D-85747 Garching, Germany,

e-mail: hhofmann@physik.tu-muenchen.de

May 27, 1999

Abstract

The linear response approach to nuclear transport has been extended to pair correlations. The latter are treated within a mean field approximation to a pairing interaction with constant matrix elements G. The constraint of particle number conservation has been accounted for on a time dependent average, which leads to modified response functions, both in the pairing degree of freedom as well as in the shape variable. The former is expressed by the gap parameter Δ and the latter by a Q which specifies the elongation of a fissioning nucleus. The tensors for friction and inertia corresponding to these two collective coordinates are computed along the fission path of ^{224}Th for temperatures around $T = 1$ MeV and less. It is seen that dissipation decreases with decreasing temperature and increasing pairing gap and falls well below the values of common "macroscopic models". Both friction and inertia show a sensible dependence on the configurations of the mean field caused both by shell effects as well as by avoided crossings of single-particle levels.

PACS numbers: -05.60.+w, 21.60.Cs, 21.60.Ev, 24.10Pa, 24.75+i

*Supported in part by the Deutsche Forschungsgemeinschaft

Contents

1 Introduction

Pair correlations are vital for understanding many elementary features of nuclear physics at zero or small thermal excitations. In particular, it is known that they have great impact on collective properties, see e.g. [1] or [2]. For the case of zero temperature this has been demonstrated in various ways. Here it may suffice to mention a few examples related to nuclear fission. Since the early 70'ties it is known that pairing will modify greatly the effective inertia [3], which in turn may change the penetrability through the barrier by orders of magnitude. In addition, the pair degree of freedom in itself may portray important features, like pair vibrations etc.[1]. A convenient way of handling such properties is by introducing the gap parameter as an additional, independent collective degree of freedom, which first has been utilized in [4]. Later this parameter has been introduced also to the generator coordinate method [5], where the influence of pairing vibrations on the spontaneous fission probability has been studied.

It is of great interest to account for pair correlations also in the description of typical transport problems of dissipative systems. On general grounds it is to be expected that pairing will greatly diminish nuclear dissipation. Indeed, there exist early formulations within linear response theory where such effects have been studied, see [6]-[7]. The present approach is meant as a generalization of these in a four fold sense: (i) We will account for particle number conservation on a time dependent average by introducing and exploiting modified response functions. (ii) We will calculate response tensors (and thus tensors for transport coefficients) for a pairing mode plus a shape degree of freedom. (iii) This will be done along a complete fission path parameterized in terms of Cassini ovaloids, thus improving the results of [8] for pair correlations. (iv) We will also address temperatures below 1 MeV, for which pair correlations become even more important.

The paper is organized as follows: In Sec. 2 we will quote basic elements of linear response theory. For a more detailed exposition of the theory we refer to the review article [9]. Sec.3 contains the derivation of the mean-field Hamiltonian and the response functions with pairing included. The Strutinsky renormalization will be applied to the calculation of the free energy in Sec. 3.2.1. In Sec.3.3 the evaluation of the collisional width for quasi-particles will be explained . In Sect.4 the modification of the response function will be given which accounts for the average particle number conservation in the dynamical process. Finally, in Sec.5 numerical results for response functions and transport coefficients are shown, the latter being compared in Sec.5.3 with those of other approaches, with special emphasis put on the temperature dependence of dissipation. Section 6 contains a short summary.

2 Linear response theory for collective motion

In this section we give a brief review of a description of collective motion within the locally harmonic approximation (LHA). It is based on the common hypothesis (see e.g. [1] and [10]) of the existence of a set of collective c-number variables Q_μ which in parametric way portray average dynamics of the nucleus as a whole. Fluctuations in these quantities shall be neglected; how the latter may be included can be found in [9]. The coupling of these collective variables to the nucleonic degrees of freedom is traced back to the deformed shell model potential which shows up in the (many body) Hamiltonian $\hat{H}_{sm}$ for independent particle motion, to be generalized later to the quasi-particle picture when

pairing correlation are to be included.

As is well known, the total energy of the system may be obtained from $\hat{H}_{sm}$ after applying Strutinsky's renormalization procedure. For later purpose it will be convenient to account for this fact by adding to the $\hat{H}_{sm}$ a c-number term $\overline{\mathcal{E}}_{pot}$ to get

$$\hat{H}_{rmf}(\hat{x}_i, \hat{p}_i, Q_\mu) = \hat{H}_{sm}(\hat{x}_i, \hat{p}_i, Q_\mu) - \overline{\mathcal{E}}_{pot} \ , \tag{1}$$

which in some sense may be called the Hamiltonian of a "renormalized mean field". At zero intrinsic excitations, the additional term can be seen to be given by the negative value of the average potential energy, see [1] and c.f. [10]. Such a construction warrants that the total energy of the system can be expressed by the mean value $\langle \hat{H}_{rmf}(\hat{x}_i, \hat{p}_i, Q_\mu) \rangle$. It may easily be extended to finite temperature and the correction term may simply be found by *requiring* that the energy obtained from the Strutinsky procedure (at finite T) be identical to the one calculated from $\hat{H}_{rmf}$, see [9].

As mentioned earlier, we will be concerned with situations of finite thermal excitations. In this case it must be expected that the *pure* independent particle model breaks down even close to the Fermi surface, in the sense that the particles (or quasi-particles) experience "collisions". Formally, the latter may be represented by adding to the $\hat{H}_{rmf}$ a *residual two-body interaction* which leads to the Hamiltonian

$$\hat{H}(\hat{x}_i, \hat{p}_i, Q_\mu) = \hat{H}_{rmf}(\hat{x}_i, \hat{p}_i, Q_\mu) + \hat{V}^{(2)}_{res}(\hat{x}_i, \hat{p}_i) \tag{2}$$

However, it is too cumbersome to work with such a form in genuine sense. For this reason we are going to approximate the effects of the $\hat{V}^{(2)}_{res}$ by dressing the single particle energies with self-energies having both real and imaginary parts. A description of details of this procedure will be deferred to a later section. We may note here, that this approximation shall be done in such a way that these self-energies are insensitive to changes in the collective coordinates Q_μ, for which reason the Q_μ have been left out in the arguments of $\hat{V}^{(2)}_{res}$.

This latter approximation goes along with the conjecture that the *"generators" of collective motion are given by the following one-body operators*

$$\hat{F}_\mu(\hat{x}_i, \hat{p}_i, Q_\mu) \equiv \frac{\partial \hat{H}(\hat{x}_i, \hat{p}_i, Q_\mu)}{\partial Q_\mu} \equiv \frac{\partial \hat{H}_{rmf}(\hat{x}_i, \hat{p}_i, Q_\mu)}{\partial Q_\mu} \tag{3}$$

These generators make up the coupling between collective and intrinsic degrees of freedom. Within the LHA this may be seen as follows, (c.f. [9]). Suppose the actual Q_μ is close to some Q^0_μ. One may then expand the effective Hamiltonian to second order as

$$\hat{H}(Q_\mu(t)) = \hat{H}(Q^0_\mu) + \sum_\mu (Q_\mu(t) - Q^0_\mu)\hat{F}_\mu + \frac{1}{2}\sum_{\mu\nu}(Q_\mu(t) - Q^0_\mu)(Q_\nu(t) - Q^0_\nu) \left\langle \frac{\partial^2 \hat{H}}{\partial Q^0_\mu \partial Q^0_\nu} \right\rangle^{\rm qs}_{Q^0_\mu, T^0} \tag{4}$$

to describe general features in the neighborhood of the Q^0_μ. Evidently, such a procedure is possible for all static quantities. In the dynamic case one needs to require that the $Q_\mu(t)$ does not move away from Q^0_μ within some time δt which is larger than the typical time for the nucleonic degrees of freedom, say the one which describes their relaxation to local equilibrium.

The generator $\hat{F}_\mu$ appearing in (4) is the one of (3) but calculated at Q^0_μ. It represents the nucleonic part of the coupling between nucleonic and collective degrees of freedom

which is linear in the latter. As implied by (4), within the LHA this is the *only* coupling term left. In the second order term the nucleonic part appears only as a static average of the corresponding operator. This average is to be built with the density operator $\hat{\rho}_{\rm qs}(Q^0_\mu)$ which in the quasi-static picture is defined by the Hamiltonian at Q^0_μ, namely $\hat{H}(Q^0_\mu)$. The $\hat{\rho}_{\rm qs}(Q^0_\mu)$ is meant to represent thermal equilibrium at Q^0_μ with the excitation being parameterized by temperature or by entropy. The simplest possibility is offered by using the canonical ensemble, or more generally, the *grand canonical distribution*. For any (static) Q_μ the latter is defined as

$$\rho_{\rm qs}(Q_\mu, T, \mu) = \frac{1}{Z'} \exp(-\hat{H}'(\hat{x}_i, \hat{p}_i, Q_\mu, \mu)/T), \quad Z' = \mathrm{tr}\,(\exp(-\hat{H}'(\hat{x}_i, \hat{p}_i, Q_\mu, \mu)/T)) \quad (5)$$

with

$$\hat{H}'(\hat{x}_i, \hat{p}_i, Q_\mu, \mu) = \hat{H}(\hat{x}_i, \hat{p}_i, Q_\mu) - \mu\hat{N} \quad (6)$$

and μ, $\hat{N}$ being the chemical potential and the particle number operator, respectively.

For the remaining part of this section we are going to restrict ourselves to a fixed N and discard complications from a possible variation of particle number. Later we are going to study the general case for which μ is considered just another "collective variable".

Before we address time dependent forces let us quote a general relation for the generalized static ones along some given direction Q_μ. With $E(Q_\mu, S)$ being the internal energy at fixed entropy S and the $\mathcal{F}(Q_\mu, T)$ the free energy at given temperature this relation reads

$$\left\langle \frac{\partial \hat{H}}{\partial Q_\nu} \right\rangle^{\rm qs} = \left(\frac{\partial \mathcal{F}(Q_\nu, T)}{\partial Q_\mu} \right)_{T, Q_{\nu\neq\mu}} = \left(\frac{\partial E(Q_\mu, S)}{\partial Q_\nu} \right)_{S, Q_{\nu\neq\mu}} \quad (7)$$

It is valid at any Q_μ both at or away from global minimum where these derivatives vanish (see [12] and [9], again).

The equations of average motion for $Q_\mu(t)$ may be constructed looking at energy conservation. For an arbitrary path through the multi-dimensional collective space the change of the total energy can be expressed as

$$0 = \frac{d}{dt} E_{\rm tot} = \sum_\mu \dot{Q}_\mu \left\langle \frac{\partial \hat{H}(\hat{x}_i, \hat{p}_i, Q)}{\partial Q_\mu} \right\rangle_t \quad (8)$$

It must vanish if we picture the nucleus as a whole as an isolated system. Requiring this condition to hold true for *any path* through the collective landscape one gets the set of equations

$$0 = \left\langle \frac{\partial \hat{H}(\hat{x}_i, \hat{p}_i, Q)}{\partial Q_\mu} \right\rangle_t = \left\langle \hat{F}_\mu \right\rangle_t + \sum_\nu (Q_\nu(t) - Q^0_\nu) \left\langle \frac{\partial^2 \hat{H}}{\partial Q^0_\mu \partial Q^0_\nu} \right\rangle^{\rm qs}_{Q^0_\mu, T^0} \quad (9)$$

To put them into convenient form one first needs to express the average $\left\langle \hat{F}_\mu \right\rangle_t$ as a functional of $Q_\mu(t)$ as well as a function of possible changes in either temperature or entropy. The form of the coupling term as given in (4) invites one to apply linear response theory to get the actual time dependent part. Considering $\sum_\mu \hat{F}_\mu (Q_\mu(t) - Q^0_\mu)$ as a time dependent perturbation one may express the deviation of the $\langle \hat{F}_\mu \rangle_t$ from their quasi-static values as (see [9])

$$\delta \langle \hat{F}_\mu \rangle_t = -\sum_\nu \int_{-\infty}^{\infty} \tilde{\chi}_{\mu\nu}(t-s)(Q_\nu(s) - Q^0_\nu) ds \quad (10)$$

where $\tilde{\chi}_{\mu\nu}$ is the causal response function

$$\tilde{\chi}_{\mu\nu}(t-s) = \Theta(t-s)\frac{i}{\hbar}\mathrm{tr}\,\left(\hat{\rho}_{\mathrm{qs}}(Q_0,T_0)[\hat{F}^I_\mu(t),\hat{F}^I_\nu(s)]\right) \equiv 2i\Theta(t-s)\tilde{\chi}''_{\mu\nu}(t-s) \qquad (11)$$

Here, the time dependence of $\hat{F}^I_\mu(t)$ (in interaction representation, if one wishes) is determined by the $\hat{H}(Q^0_\mu)$. The Fourier transform of (10) reads

$$\delta\langle\hat{F}_\mu\rangle_\omega = -\sum_\nu \chi_{\mu\nu}\delta Q_\nu(\omega) \qquad \text{with} \qquad \delta Q_\nu(\omega) = Q_\nu(\omega) - Q^0_\nu\delta(\omega) \qquad (12)$$

with $\delta Q_\nu(\omega)$ being the Fourier transform of $(Q_\nu(s) - Q^0_\nu)$.

Piecing together all relevant parts, in Fourier representation and exploiting vector notation the set of equations of motion may finally be written in the following compact form

$$\delta\langle\hat{\mathbf{F}}\rangle_\omega = \mathbf{k^{-1}q}(\omega) \qquad \text{with} \qquad \mathbf{q} = \mathbf{Q} - \mathbf{Q}_m \qquad (13)$$

and where the Q^m_μ defines the center of the local oscillators (see [9] where all details have been worked out carefully for the one-dimensional case). In (13) a tensor $k_{\mu\nu}$ appears whose elements often are interpreted as coupling constants, see below. At zero excitation it would simply be given by the negative value of the average of the second derivatives of the Hamiltonian [10]. At finite excitations additional terms show up which involve static susceptibilities. Which ones will appear depends on the situation one chooses. In [9] it has been argued in favor of working at constant entropy. It is much simpler, though, to assume that temperature does not change within the time lapse δt for which the equations of collective motion are derived. For the sake of simplicity we will adhere to the latter case, knowing that in this respect we are along the lines of most of the papers in this field. For such a situation the coupling tensor, which often is expressed by its inverse $\boldsymbol{\kappa} \equiv \boldsymbol{k}^{-1}$, is given by (c.f. [9])

$$-\left(k^{-1}\right)_{\mu\nu} \equiv -\kappa_{\mu\nu} = \left\langle\frac{\partial^2\hat{H}}{\partial Q_\mu\partial Q_\nu}\right\rangle^{\mathrm{qs}} + \chi_{\mu\nu}(0) - \chi^{\mathrm{T}}_{\mu\nu} = \frac{\partial^2\mathcal{F}(Q,T)}{\partial Q_\mu\partial Q_\nu} + \chi_{\mu\nu}(0) \qquad (14)$$

Here, $\chi_{\mu\nu}(0)$ is the static response and

$$\chi^{\mathrm{T}}_{\mu\nu} = -\frac{\partial\langle\hat{F}_\mu\rangle^{\mathrm{qs}}}{\partial Q_\nu}\bigg|_T \qquad (15)$$

the isothermal susceptibility defined as the change of the quasi-static expectation value $\langle\hat{F}_\mu\rangle^{\mathrm{qs}}$ with Q_ν at given T. It may be noted that at lower temperature the difference $\chi_{\mu\nu}(0) - \chi^{\mathrm{T}}_{\mu\nu}$ — known to vanish exactly at $T = 0$ — may often be neglected as compared to the first term on the right of (14). Indeed, such a case will be encountered below.

Finally, we aim at equations of motion for the Q_μ of a structure similar to that of Newton's equation with a dissipative force. To be able identifying uniquely all forces, as well as the corresponding coefficients for inertia, dissipation and local stiffness, it is useful to introduce a coupling $U^{ext}(t)$ to some "external" fields $f^{ext}_\mu(t)$ which finally will show up as in-homogeneous terms. A convenient form is given by

$$U^{ext}(t) = \sum_\mu \hat{F}_\mu f^{ext}_\mu(t) \qquad (16)$$

Defining a response tensor $\boldsymbol{\chi}_{\rm coll}(\omega)$ by

$$\delta\langle\hat{F}_\mu\rangle_\omega = -\sum_\nu \chi^{\rm coll}_{\mu\nu}(\omega) f^{ext}_\nu(\omega) \tag{17}$$

it can be shown (see [11] and [9], for instance, or [10] for the undamped case at $T = 0$) to attain the form

$$\boldsymbol{\chi}_{\rm coll}(\omega) = \boldsymbol{\kappa}(\boldsymbol{\kappa} + \boldsymbol{\chi}(\omega))^{-1}\boldsymbol{\chi}(\omega) \tag{18}$$

Its pole structure is determined by the collective excitations of the system, whence the name "collective" response tensor comes from.

To finally be able introducing the transport coefficients for average collective motion, as expressed in terms of the $q_\mu(t)$, one first rewrites (17) in the form

$$[\boldsymbol{\kappa}\boldsymbol{\chi}_{\rm coll}{}^{-1}(\omega)\boldsymbol{\kappa}]\boldsymbol{q}(\omega) = -\boldsymbol{\kappa}\boldsymbol{f}^{ext}(\omega) \equiv -\boldsymbol{q}^{ext}(\omega) \tag{19}$$

Here, (13) has been used and the external fields q^{ext} for the q_μ-modes are defined through the last equation on the very right. The set (19) turns into the conventional form of Newtonian dynamics

$$\sum_\nu (M_{\mu\nu}\ddot{q}_\nu(t) + \gamma_{\mu\nu}\dot{q}_\nu(t) + C_{\mu\nu}q_\mu(t)) = -q^{ext}_\mu(t) \tag{20}$$

if one may approximate the quantity $\boldsymbol{\kappa}\boldsymbol{\chi}_{\rm coll}{}^{-1}(\omega)\boldsymbol{\kappa}$ by a second order polynomial in frequency

$$(\boldsymbol{\kappa}\boldsymbol{\chi}_{\rm coll}{}^{-1}(\omega)\boldsymbol{\kappa})_{\mu\nu} \quad \Longrightarrow \quad -M_{\mu\nu}\omega^2 - i\gamma_{\mu\nu}\omega + C_{\mu\nu} \tag{21}$$

Evidently, the coefficients $M_{\mu\nu}$, $\gamma_{\mu\nu}$ and $C_{\mu\nu}$ stand for the elements of the tensors for mass, friction and stiffness.

Without any doubt, the transition (21) is crucial to the question as to which extent Markovian type of equations may be justified for the very complex situation of nuclear collective motion. In the past, two version have been worked out for this transition (see [9] for a review and a guide to original work). On the one hand, one may study the collective strength distribution as given by the dissipative part of the collective response. Whenever this distribution has prominent peaks (for the multi-dimensional case see [11]) they may be fitted by Lorentzians, which in turn may be interpreted as the oscillator response function associated to the form (20) of the classical equations of motion. Another possibility is offered by simply expanding the functions given on the left hand side of (21) to second order in ω around $\omega = 0$. In the one-dimensional case one gets, see [8]),

$$C \approx \frac{1}{k^2\chi_{\rm coll}(\omega)}\Big|_{\omega=0} = \frac{\chi(0)+C(0)}{\chi(0)}C(0) \tag{22}$$

$$\gamma \approx \frac{1}{k^2}\frac{\partial(\chi_{\rm coll}(\omega))^{-1}}{\partial\omega}\Big|_{\omega=0} = \frac{(\chi(0)+C(0))^2}{\chi^2(0)}\gamma(0) \tag{23}$$

and

$$M \approx \frac{1}{2k^2}\frac{\partial^2(\chi_{\rm coll}(\omega))^{-1}}{\partial\omega^2}\Big|_{\omega=0} = \frac{(\chi(0)+C(0))^2}{\chi^2(0)}\left(M(0) + \frac{\gamma^2(0)}{\chi(0)}\right) \tag{24}$$

The friction $\gamma(0)$ and mass parameters $M(0)$ are expressed in terms of first and second derivatives of the response function at $\omega = 0$. Their modification to the multi dimensional case are of the forms

$$\gamma_{\mu\nu}(0) = -i\frac{\partial\hat{\chi}_{\mu\nu}(\omega)}{\partial\omega}\Big|_{\omega=0} = \frac{\partial\hat{\chi}''_{\mu\nu}(\omega)}{\partial\omega}\Big|_{\omega=0} \tag{25}$$

and

$$M_{\mu\nu}(0) = \frac{1}{2}\frac{\partial^2 \hat{\chi}_{\mu\nu}(\omega)}{\partial\omega^2}\Big|_{\omega=0} = \frac{1}{2}\frac{\partial^2 \hat{\chi}'_{\mu\nu}(\omega)}{\partial\omega^2}\Big|_{\omega=0}. \tag{26}$$

For obvious reasons, expressions (25),(26) may be referred to as the "zero frequency limit".

3 The inclusion of pairing degrees of freedom

In this section we are going to address pair correlations. This shall be done in two steps. At first we concentrate on the pairing mode alone to subsequently address the general case of pairing plus a shape degree of freedom.

3.1 The conventional pairing model

The generator for the pairing mode may be defined as the following pairing field operator $\hat{P}^\dagger$

$$\hat{P}^\dagger = \sum_k a_k^\dagger a_{\bar{k}}^\dagger. \tag{27}$$

The $a_k^\dagger$ and a_k are the creation and annihilation operators for the normal vacuum. The summation over single-particle states k is meant not to include the time reversed ones, which are denoted by $\bar{k}$. Commonly one does not start with the associated mean field Hamiltonian, as we did in the first section, but with a separable two body interaction instead, which may be constructed from the $\hat{P}$ and $\hat{P}^\dagger$. The corresponding many body Hamiltonian may thus be written as

$$\hat{H}^{\mathcal{P}} = \hat{H}_{sm} - G\hat{P}^\dagger\hat{P} \tag{28}$$

where the coefficient G of the two body interaction, the coupling constant, stands for a constant pairing matrix. The first part $\hat{H}_{sm}$ is meant to represent motion of *independent particles*, and hence may be written as

$$\hat{H}_{sm} = \sum_k \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) \tag{29}$$

with the ϵ_k being the single-particle energies. How this $\hat{H}_{sm}$ relates to the shape degrees of freedom shall be of no concern for the moment. As indicated earlier this question will be addressed later.

To establish relations to the discussion in the previous section let us introduce the mean field approximation to (28) by writing

$$\hat{H}^{\mathcal{P}} = \hat{H}_{sm} - \Delta(\hat{P}^\dagger + \hat{P}) + \frac{\Delta^2}{G} + \hat{H}_{res} \tag{30}$$

with

$$\Delta \equiv G\langle\hat{P}\rangle = G\langle\hat{P}^\dagger\rangle \tag{31}$$

and

$$\hat{H}_{res} = -G\sum_{kj}(a_k^\dagger a_{\bar{k}}^\dagger - \langle a_k^\dagger a_{\bar{k}}^\dagger\rangle)(a_{\bar{j}}a_j - \langle a_{\bar{j}}a_j\rangle) \tag{32}$$

Nothing else has been done but rewriting (28) with the help of the c-number variable Δ. Actually at this stage it may be unclear yet whether or not the averages of P and $P^\dagger$ are identical. In the worst case we might simply introduce a complex Δ. However, as we shall see a real one will do.

Evidently, the mean field part of (30) may be identified as

$$\hat{H}^{\mathcal{P}}_{rmf} = \hat{H}_{sm} - \Delta(\hat{P}^\dagger + \hat{P}) + \frac{\Delta^2}{G} \tag{33}$$

Actually, in spirit of the last section the word "renormalized" is appropriate here; it is justified because of the c-number term on the very right. Indeed, this term is necessary in order to have the expectation value of $\hat{H}^{\mathcal{P}}_{rmf}$ represent the total energy, see e.g. sect.3.1.5 of [9], in particular eq.(3.1.45). For the present model this c-number term is nothing else but the $\overline{\mathcal{E}}_{pot}$ of (1). In summary, the mean field approximation to (28) simply implies to neglect the residual interaction H_{res}; see also [7] or [15].

One might be inclined to associate the $\hat{H}_{res}$ of (32) with the $\hat{V}^{(2)}_{res}$ introduced in (2). This is not the appropriate connection, however, as the $\hat{V}^{(2)}_{res}$ is meant to simulate collisional damping. There can be little doubt that it would be asking too much if one were to deduce the latter from a separable interaction of the type given in (28). Indeed, later on we are going to take into account this latter type of residual interaction, albeit in special form, namely through self-energies, which in some sense may be considered a generalized mean field approximation.

The Δ introduced in (30) will henceforth be considered the collective variable for the pairing degree of freedom and the associated mean field Hamiltonian will be the one of (33). The alert reader may have noticed some similarity of (31) with (13), indicating a possible connection of the G to the coupling constant k introduced in the previous section; this latter relation will be worked out in more detail below. Notice, however, an essential difference between (13) and (31). Whereas the former case refers to a time dependent situation in which the expectation values have a definite meaning. Those appearing in (31), on the contrary, are not yet fully specified. Indeed, the Hamiltonian given in (33) is of practical use only if the Δ may at first be treated as a *free parameter — as it ought to be for a collective variable — discarding this subsidiary condition (31).* It is only after the Δ has been assigned a special value (which for the static case will be that of global equilibrium), or a well defined function of time (as in the dynamic case discussed above for the general situation, with its specification to the case of pairing to come below) that these expectation values will be defined properly.

3.1.1 Transformation to quasi-particles

In the following we are going to diagonalize the Hamiltonian $\hat{H}^{\mathcal{P}}_{rmf}$ by transforming to quasi-particles in strict sense. In doing so we will not make use of the subsidiary condition (31). The relations to be discussed now will hold true in the more general sense, not just at equilibrium which we will look at below. Actually it is at this place that we exploit the fact of Δ being a free parameter.

When accounting for pair correlations it is difficult to conserve particle number. For this reason we will refer to the *grand canonical ensemble* introduced in (5) and (6). This means to replace the $\hat{H}^{\mathcal{P}}_{rmf}$ by

$$\hat{H}^{\mathcal{P}\,\prime}_{rmf} = \hat{H}^{\mathcal{P}}_{rmf} - \mu\hat{N} \tag{34}$$

fixing the Lagrange multiplier μ by requiring particle number conservation on average, viz

$$\langle \hat{N} \rangle = N \qquad \text{with} \qquad \hat{N} = \sum_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) \tag{35}$$

To diagonalize the Hamiltonian $\hat{H}^{\mathcal{P}\,\prime}_{rmf}$ let us perform first the Bogolyubov transformation

$$a_k = u_k \alpha_k + v_k \alpha_{\bar{k}}^\dagger \qquad a_{\bar{k}} = u_k \alpha_{\bar{k}} - v_k \alpha_k^\dagger \tag{36}$$

to new operators $\alpha_k, \alpha_k^\dagger \ldots$ which themselves are required to satisfy the anti-commutation rules of Fermions. As one knows, this implies the constraints

$$u_k^2 + v_k^2 = 1. \tag{37}$$

Next we write

$$\hat{H}^{\mathcal{P}\,\prime}_{rmf} = U_0' + H_{11}' + H_{20}' \tag{38}$$

where

$$\begin{aligned} U_0' &= \sum_k 2v_k^2(\epsilon_k - \mu) - 2\Delta \sum_k u_k v_k + \frac{\Delta^2}{G} \ , \\ H_{11}' &= \sum_k [(\epsilon_k - \mu)(u_k^2 - v_k^2) + 2\Delta u_k v_k)](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \ , \\ H_{20}' &= \sum_k [(\epsilon_k - \mu) 2u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k) \end{aligned} \tag{39}$$

following common notation. To get from (38)-(39) the independent quasi-particle Hamiltonian we must chose the coefficients u_k and v_k of the transformation (36) to make the last term vanish: $H_{20}' = 0$. As it is seen from (39), this condition has the form

$$(\epsilon_k - \mu) 2u_k v_k - \Delta(u_k^2 - v_k^2) = 0 \tag{40}$$

This equation, together with normalization condition (37), can be solved as

$$u_k^2 = \frac{1}{2}\left(1 + \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}}\right), \qquad v_k^2 = \frac{1}{2}\left(1 - \frac{\epsilon_k - \mu}{\sqrt{(\epsilon_k - \mu)^2 + \Delta^2}}\right) \tag{41}$$

Finally, the $\hat{H}^{\mathcal{P}\,\prime}_{rmf}$ becomes

$$\hat{H}^{\mathcal{P}\,\prime}_{rmf} = U_0' + \sum_k E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \tag{42}$$

with

$$E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}. \tag{43}$$

3.1.2 Thermodynamics of quasi particles

Having the quasi-particle Hamiltonian $\hat{H}^{\mathcal{P}\,\prime}_{rmf}$ of (42) at our disposal we may now look at the density operator for the *quasi-static* equilibrium for the pairing degrees of freedom. It is a special case of (5) it reads

$$\rho_{\rm qs}(\Delta, T) = \frac{1}{Z'} \exp(-\hat{H}^{\mathcal{P}\,\prime}_{rmf}/T) \qquad\qquad Z' = \text{tr } \exp(-\hat{H}^{\mathcal{P}\,\prime}_{rmf}/T) \tag{44}$$

(Please observe that the Δ still is to be considered a free parameter). For a Hamiltonian like the one of (42) representing independent motion of quasi-particles it is not difficult to calculate various quantities of interest. For the grand potential one gets

$$\Omega = -T \log Z' = \sum_k (\epsilon_k - \mu - E_k) - 2T \sum_k \log(1 + \exp(-E_k/T)) + \frac{\Delta^2}{G}\,, \tag{45}$$

and for the particle number

$$N = -\frac{\partial \Omega}{\partial \mu} = 2 \sum_k n_k \tag{46}$$

where

$$n_k = \langle a_k^\dagger a_k \rangle^{\rm qs} = v_k^2 + (u_k^2 - v_k^2) n_k^T = \frac{1}{2}\left(1 - \frac{\epsilon_k - \mu}{E_k} \tanh \frac{E_k}{2T}\right), \tag{47}$$

with

$$n_k^T = \langle \alpha_k^\dagger \alpha_k \rangle^{\rm qs} = (1 + \exp(E_k/T))^{-1} \tag{48}$$

being the probability that the state k is excited. Let $\mathcal{P}$ be the average value of the pairing field operator for the ensemble (44); one finds

$$\mathcal{P} \equiv \langle \hat{P} \rangle^{\rm qs} \equiv \langle \hat{P}^\dagger \rangle^{\rm qs} = \sum_k \phi_k \tag{49}$$

with

$$\phi_k = \langle a_k^\dagger a_{\bar{k}}^\dagger \rangle^{\rm qs} = u_k v_k (1 - 2n_k^T) = \frac{\Delta}{2E_k}(1 - 2n_k^T). \tag{50}$$

The internal energy may be written as

$$E = \langle \hat{H}_{rmf}^{\mathcal{P}\,\prime} \rangle^{\rm qs} + \mu N \equiv \langle \hat{H}_{rmf}^{\mathcal{P}} \rangle^{\rm qs} = 2 \sum_k \epsilon_k n_k - 2\Delta \mathcal{P} + \frac{\Delta^2}{G} \tag{51}$$

and for the free energy one gets

$$\mathcal{F} = \Omega + \mu N = 2 \sum_k \epsilon_k n_k - 2\Delta \mathcal{P} + \frac{\Delta^2}{G} - TS \tag{52}$$

with the entropy S being given by

$$S = 2 \sum_k s_k \qquad \text{with} \qquad s_k = [n_k^T \log n_k^T + (1 - n_k^T) \log(1 - n_k^T)] \tag{53}$$

At various places we need the derivatives with respect to Δ rather than the energies themselves. In principle, they could be obtained from (51) and (52). It is much easier, however, to exploit eq.(7). For the pairing degree of freedom, and for the present purpose, the appropriate Hamiltonian to be used there is the $\hat{H}_{rmf}^{\mathcal{P}}$ of (33). Obviously, its derivative is given by

$$\frac{\partial \hat{H}_{rmf}^{\mathcal{P}}(\Delta)}{\partial \Delta} = -(\hat{P} + \hat{P}^\dagger) + \frac{2\Delta}{G} \tag{54}$$

Thus (7) leads to

$$\left(\frac{\partial \mathcal{F}(Q, \Delta, T)}{\partial \Delta}\right)_{T,Q} = -2\mathcal{P} + \frac{2\Delta}{G} = \left(\frac{\partial E(Q, \Delta, S)}{\partial \Delta}\right)_{S,Q} \tag{55}$$

It is reassuring to see that the same result may be obtained starting from the expression for the free energy given in (52), indeed.

Obviously, this force vanishes at

$$\mathcal{P}^{\rm eq} \equiv \mathcal{P}(\Delta^{\rm eq}) = \frac{\Delta^{\rm eq}}{G} \tag{56}$$

thus defining the value $\Delta^{\rm eq}$ at which either the internal energy at fixed entropy or the free energy at fixed temperature become minimal. Notice that (56) is an *implicit equation* for $\Delta^{\rm eq}$ with the functional form of $\mathcal{P}(\Delta)$ being given by (49), together with (50) and the expressions given before for E_k as well as for the thermal occupation number: Eq.(56) is nothing else but the *gap equation* at finite temperature, which can easily be brought to the conventional form

$$\frac{2}{G} = \sum_k \frac{1-2n_k^T}{E_k} = \sum_k \frac{\tanh(E_k/(2T))}{E_k} \tag{57}$$

As one may recognize, the relation (56) is of the form given in (31), which in this context has obtained a specific meaning. For a static situation, the averages appearing in (31) are those of thermodynamic equilibrium, in which case the subsidiary condition (31) is valid for Δ replaced by $\Delta^{\rm eq}$. As this special form is obtained after minimizing the total energy, one may understand why sometimes it is referred to as the "self-consistency condition" (for the separable interaction we started with).

3.1.3 Remarks on the coupling constant

In (14) a coupling constant k has been introduced which depends on the collective variables as well as on excitation. Let us see how this k relates to the G of (28). A direct application of (14) leads to

$$-\frac{1}{k_\Delta} = \left\langle \frac{\partial^2 \hat{H}^{\mathcal{P}}_{rmf}(\Delta)}{\partial \Delta^2} \right\rangle^{\rm qs} + (\chi_{PP}(\omega=0) - \chi^T_{PP}) = \frac{2}{G} + (\chi_{PP}(\omega=0) - \chi^T_{PP}) \tag{58}$$

For the second equation we have simply made use of (54). The response function and the isothermal susceptibility appearing there may be traced back to (11) and (15) if only the $\hat{F}_\mu$ is replaced by $-(\hat{P}+\hat{P}^\dagger)$. Within the independent quasi-particle model their calculation is straightforward. (The evaluation of the response in the more general context will be given below, in particular in the Appendix). As the susceptibility may be written as $\chi^T_{PP} = 2\partial\mathcal{P}/\partial\Delta$ it may be obtained by differentiating (49) with respect to Δ keeping temperature and particle number fixed. One gets

$$\chi^T_{PP} = \widehat{\chi}_{PP}(0) - 2\sum_k \frac{\Delta}{E_k}\frac{\partial n_k^T}{\partial \Delta} \tag{59}$$

(The $\widehat{\chi}_{PP}(\omega)$ is the response function calculated for fixed particle number, for details see below). Estimating numerically the difference $\chi^T_{PP} - \chi_{PP}(\omega=0)$ from (59) it is seen to be negligibly small as compared to the leading term $2/G$. Even at $T = 2$ MeV it makes up less than 10%. Exploiting this fact we may conclude

$$-k_\Delta \approx G/2\,, \qquad \text{to put} \qquad -k_\Delta \equiv G/2 \tag{60}$$

Evidently, for this comparison of k_Δ and G, there is no need to worry about the different signs or the appearance of the factor of 2, which are simply due to the fact of using in (28) a form commonly found in the literature. However, a few words might be in order to explain why this relation has turned out not to be an exact one. First of all, we may recall, that at $T = 0$ this would be so in strict sense. Secondly, it had been mentioned already when discussing the appearance of the k in (13), that for the nucleus as an isolated system a more appropriate form would be the one in which in (14) the isothermal susceptibility gets replaced by the adiabatic one $\chi^{\rm ad}$ defined for fixed entropy. This difference, $\chi^{\rm ad}-\chi(0)$, is known to vanish identically for ergodic systems, see [9], in which case $-k_\Delta$ equals $G/2$ exactly, indeed. Thirdly, it can be said for *diabatic motion of independent quasi-particles* this difference again would vanish identically. This observation follows from the fact for such a model diabatic motion is given for *fixed occupation numbers* in which case the n_k^T do not vary with the collective variables (like the Δ in the present case).

3.2 The general mean field for pairing and shape variables

After this detour into the peculiarities of the pairing model we may now go back to the starting point in Sec. 2 to combine the collective variable Δ for pairing with those for the shape. Actually, to keep the numerical effort on a manageable level, we restricted the computations to just one shape variable, the elongation parameter $Q = R_{12}/2R_0$ which describes the distance between the left and right centers of mass. To have a dimensionless quantity this distance is commonly divided by the diameter of the sphere of identical volume, see [8].

To construct the Hamiltonian $\hat{H}_{sm}(\hat{x}_i, \hat{p}_i, Q_\mu)$ of (1) for the set $Q_\mu \equiv Q, \Delta$, we simply need to choose for the $\hat{H}_{sm}$ of (33) the Hamiltonian $\hat{H}_{sm}(\hat{x}_i, \hat{p}_i, Q)$ of the deformed shell model. The quasi-static density operator for the grand canonical ensemble is given by (5). The generators of collective motion defined in (3), specialized now to $Q_\mu \equiv Q, \Delta$, turn out to be as follows. The one for the pairing mode is given already by (54). Let us write it as

$$\hat{F}_\Delta = \frac{\partial \hat{H}_{rmf}(Q,\Delta)}{\partial \Delta} = -(\hat{P} + \hat{P}^\dagger) + \frac{2\Delta}{G} \tag{61}$$

It explicitly contains the c-number correction $2\Delta/G$. The equivalent form for the shape degree of freedom reads

$$\hat{F}_Q = \frac{\partial \hat{H}_{rmf}(Q,\Delta)}{\partial Q} = \frac{\partial \hat{H}_{sm}(Q,\Delta)}{\partial Q}\Big|_{Q=Q_0} - \frac{\partial \overline{\mathcal{E}}_{pot}}{\partial Q}\Big|_{Q=Q_0} \equiv \hat{F} - \frac{\partial \overline{\mathcal{E}}_{pot}}{\partial Q}\Big|_{Q=Q_0} \tag{62}$$

where the last term in the end would have to be calculated through the Strutinsky renormalization, see below. It should be noted however, that these corrections may be omitted when calculating the response functions. They simply drop out of the commutators in (11).

Next we turn to the coupling constants (14). Again, following the discussion of (60) the one for the pairing degree is solely determined by G, in that we have $-k_\Delta \equiv G/2 \equiv -1/\kappa_\Delta$. The one for the shape is given by

$$-\kappa_Q = \frac{\partial^2 \mathcal{F}(Q,\Delta,T)}{\partial Q^2} + \chi_{FF}(\omega = 0) \tag{63}$$

This one will depend both on our collective variables Q and Δ as well as on temperature T. In principle, there could be a non-diagonal one. Following strictly the formulas given above we would find

$$-\kappa_{Q\Delta} = \frac{\partial^2 \mathcal{F}(Q,\Delta,T)}{\partial Q \partial \Delta} - \chi_{FP}(\omega=0) = \chi^T_{FP} - \chi_{FP}(\omega=0) \tag{64}$$

The expression on the very right of (64) follows from (55) once its Q-dependence is taken into account. We find it wiser, however, to simply put this quantity equal to zero, essentially for two reasons, beyond that of simplicity: 1) Like before for $\Delta\Delta$ case, the difference of the susceptibilities $\chi^T_{FP} - \chi_{FP}(\omega=0)$ can be expected to be small at small T. 2) Again, we are faced with the fact that the shape and the pairing degrees of freedom are not really based on the same footing — which by the way is a feature common to almost all microscopic approaches. On the one hand, for the pairing degree of freedom there is no mean field at our disposal which might be considered as realistic as that for the shape degrees of freedom (completed by the Strutinsky renormalization). As a matter of fact, it was constructed from a separable two body interaction. 3) Conversely, for the shape degrees of freedom it would not make much sense to start from a similar schematic force. We simply believe our approximating the mean field in this case to be better.

3.2.1 Strutinsky renormalization

Let us address the calculation of the free energy $\mathcal{F}$, needed for instance for the coupling constant κ_Q given in (63). One might exploit the expression (52) derived within independent quasi-particles model. As is well known, such an approximation holds true only in the neighborhood of the Fermi level. But such deficiencies can easily be overcome by applying the Strutinsky renormalization procedure [23, 24]. Thus we represent $\mathcal{F}$ as the sum of the liquid drop part and the shell correction

$$\mathcal{F} = \mathcal{F}_{LDM} + \delta\mathcal{F} \tag{65}$$

Unfortunately, we can not use here the standard expression for the shell correction to the *ground state energy* given in [24]. The derivation of this expression rely very much on the gap equation which does not hold away from equilibrium with respect to Δ. For an arbitrary Δ we should proceed in slightly different way. Namely, we may make use of the advantage of Strutinsky's energy theorem according to which the shell correction to the total energy of the system of interacting particles can be obtained within a mean field approximation. In our case it means that we can use the free energy derived not with the Hamiltonian (28) but with independent quasi-particles Hamiltonian (42). Namely, we will define the shell correction to the free energy as

$$\delta\mathcal{F} = \mathcal{F} - \widetilde{\mathcal{F}} \tag{66}$$

where $\mathcal{F}$ is given by (52) and $\widetilde{\mathcal{F}}$ is its average.

The numerical computation of the average free energy $\mathcal{F}$ is done in two steps. Firstly, in eqs.(52) we substitute summation over the single-particle states by integration over the single-particle energies with their density of states being that of the independent particle model

$$\sum_k \to \int_{-\infty}^{\infty} g_s(e)de \quad \text{where} \quad g_s(e) = \sum_k \delta(\epsilon_k - e) \tag{67}$$

In a second step we replace the density $g_s(e)$ by the smoothed quantity

$$g_s(e) \to \widetilde{g}(e) = \frac{1}{\gamma_{\rm av}} \int_{-\infty}^{\infty} g_s(e') f_{av}\left(\frac{e'-e}{\gamma_{\rm av}}\right) de' = \frac{1}{\gamma_{\rm av}} \sum_k f_{av}\left(\frac{\epsilon_k - e}{\gamma_{\rm av}}\right) \tag{68}$$

where $f_{av}(x)$ is the smoothing function of the shell correction method, see [23, 24]. In this way the smoothed free energy becomes

$$\widetilde{\mathcal{F}} = 2\sum_k \epsilon_k \widetilde{n}_k - 2\Delta \sum_k \widetilde{\phi}_k + \frac{\Delta^2}{G} - 2T \sum_k \widetilde{s}_k \tag{69}$$

The averaged quantities $\widetilde{n}_k$, $\widetilde{s}_k$ and $\widetilde{\phi}_k$ are given by (see also [25])

$$\begin{aligned}
\widetilde{n}_k &= \widetilde{n}(\epsilon_k) = \frac{1}{\gamma_{\rm av}} \int_{-\infty}^{\infty} n(e) f_{av}\left(\frac{e-\epsilon_k}{\gamma_{\rm av}}\right) de \\
\widetilde{s}_k &= \widetilde{s}(\epsilon_k) = \frac{1}{\gamma_{\rm av}} \int_{-\infty}^{\infty} s(e) f_{av}\left(\frac{e-\epsilon_k}{\gamma_{\rm av}}\right) de \\
\widetilde{\phi}_k &= \widetilde{\phi}(\epsilon_k) = \frac{1}{\gamma_{\rm av}} \int_{-\infty}^{\infty} \phi(e) f_{av}\left(\frac{e-\epsilon_k}{\gamma_{\rm av}}\right) de
\end{aligned} \tag{70}$$

With these quantities at our disposal we may express the first order shell correction $\delta\mathcal{F}$ as

$$\delta\mathcal{F}(\Delta, T) = 2\sum_k (\epsilon_k \delta n_k - \Delta \delta\phi_k - T\delta s_k) \tag{71}$$

with $\delta n_k = n_k - \widetilde{n}_k$, etc. In the vicinity of the Fermi energy the quantities n_k, ϕ_k and

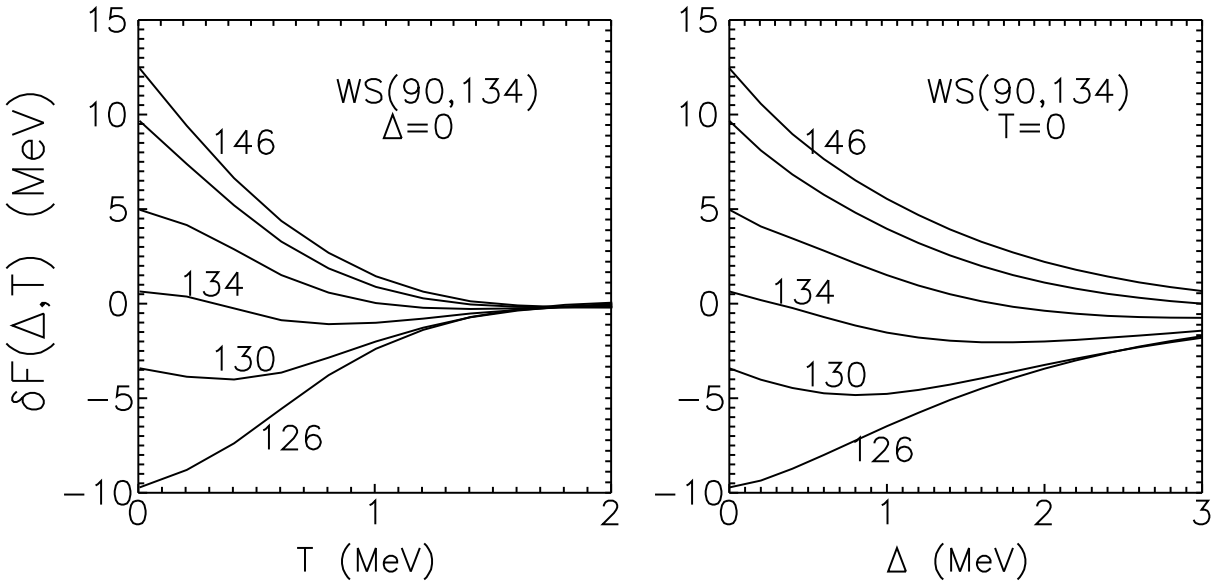

Figure 1: The shell correction to the free energy (71) for neutrons in the spherical Woods-Saxon potential as function of temperature for $\Delta = 0$ (left-hand-side) and as function of Δ for $T = 0$ (right-hand-side). The neutron numbers are indicated in the figure. The parameters of the Strutinsky smoothing procedure are: averaging interval $\gamma_{\rm av} = 8$ MeV, order of the curvature polynomial: $M = 6$.

s_k are strongly varying functions of ϵ_k, but they behave rather smoothly far away from it. As a consequence, the δn_k, $\delta\phi_k$ and δs_k differ substantially from zero only for states close to the Fermi energy. This implies that the sums in (71) are rather insensitive to a variation of their limits. In particular, the problem of logarithmic convergence in the solving gap equation does not appear in (71). Thus one need not restrict the summation in (71) to the so called pairing window.

Fig.1 shows the dependence of the shell correction (71) as function of temperature for $\Delta = 0$ and as function of Δ for $T = 0$. It is seen that $\delta\mathcal{F}(\Delta, T)$ decreases both with temperature as well as with Δ. The feature may be understood as follows. With growing T or Δ, the occupation numbers n_k of the states close to the Fermi energy become more and more diffuse and thus resemble more the average value $\widetilde{n}_k$. The same holds true for the s_k and ϕ_k. Consequently, the absolute values of the δn_k etc. decrease and, hence, the sum (71) becomes smaller. The temperature at which the shell structure disappears is approximately equal to 1.5 MeV. Here, the shell correction makes up only a few percent of its value at $T = 0$. The decrease of the shell correction with the pairing gap (at fixed temperature) is less rapid. Even for $\Delta = 2$ MeV the shell correction still amounts to a few dozen percent of its value at $\Delta = 0$.

3.2.2 Intrinsic response function for independent quasi-particles

The time-dependent $\mu\nu$-response functions for "intrinsic" or nucleonic motion are to be calculated starting from the definition (11). To begin with we first address the model of independent quasi-particles, discarding for the moment the modifications necessary for "collisional damping", which will be discussed below. Moreover, we want to concentrate on the FF function. The results for the FP and the PP response are given in the Appendix.

In the quasi-particle representation the (single-particle) operator $\hat{F}$ can be written as

$$\hat{F} = \sum_k F_{kk} 2v_k^2 + \sum_{kj} F_{kj}\xi_{kj}(\alpha_k^\dagger\alpha_j + \alpha_{\bar{k}}^\dagger\alpha_{\bar{j}}) + \sum_{kj} F_{kj}\eta_{kj}(\alpha_k^\dagger\alpha_{\bar{j}}^\dagger + \alpha_{\bar{j}}\alpha_k) \tag{72}$$

where

$$\eta_{kj} \equiv u_k v_j + v_k u_j\,, \quad \xi_{kj} \equiv u_k u_j - v_k v_j \tag{73}$$

Inserting (72) into (11) after somewhat lengthy but straightforward calculations one gets

$$\begin{aligned}\tilde{\chi}_{FF}(t) = &\frac{-2\theta(t)}{\hbar}{\sum_{kj}}'(n_k^T - n_j^T))\xi_{kj}^2|F_{kj}|^2\sin((E_k - E_j)t/\hbar) \\ &-\frac{2\theta(t)}{\hbar}\sum_{kj}(n_k^T + n_j^T - 1)\eta_{kj}^2|F_{kj}|^2\sin((E_k + E_j)t/\hbar)\end{aligned} \tag{74}$$

Notice please, that in the first sum no diagonal components of the "ξ"-terms survive. In the end this is due to the fact that the corresponding terms of the operator $\hat{F}$ commute with the Hamiltonian (38) and do not contribute to the response function (74). That is why the first sum in (74) is marked by a prime. In the second sum of (74) both diagonal and non-diagonal components contribute. The Fourier transform of (74) lead to

$$\chi_{FF}(\omega) = {\sum_{kj}}'(n_k^T - n_j^T)\xi_{kj}^2\left(\frac{1}{\hbar\omega - (E_k - E_j) + i\varepsilon} - \frac{1}{\hbar\omega + (E_k - E_j) + i\varepsilon}\right)|F_{kj}|^2$$

$$+\sum_{kj}(n_k^T+n_j^T-1)\eta_{kj}^2\left(\frac{1}{\hbar\omega-(E_k+E_j)+i\varepsilon}-\frac{1}{\hbar\omega+(E_k+E_j)+i\varepsilon}\right)|F_{kj}|^2 \quad (75)$$

As it is seen from the last equation, the first line of (75) does not contribute to the response function for $T=0$ since in this case *all* $n_k^T=0$. For nonzero temperature both sums of (75) do contribute. Please observe that the second line only represents contributions from excitations above 2Δ, as $E_k+E_j\geq 2\Delta$. Conversely, as E_k-E_j may be arbitrarily small, contributions to the truly low frequency region may only come from the first line in (75). Apparently they may built up only at finite temperatures.

Eq.(74) can be brought to the more compact form

$$\tilde{\chi}_{FF}(t)=\frac{-\theta(t)}{\hbar}\{\sum_{k=j}\sum_{s\neq s'}+\sum_{k\neq j}\sum_{ss'}\}(n_{ks}^T-n_{js'}^T)\xi_{ksjs'}^2|F_{kj}|^2\sin\left(\frac{E_{ks}-E_{js'}}{\hbar}\right)t \quad (76)$$

by introducing both positive as well as negative quasi-particle energies

$$E_{ks}=s\sqrt{(\epsilon_k-\mu)^2+\Delta^2}\,,\ \text{with}\ s=\pm 1 \quad (77)$$

together with the corresponding amplitudes u_{ks} and v_{ks}

$$u_{ks}=\frac{1}{\sqrt{2}}\left(1-\frac{\epsilon_k-\mu}{E_{ks}}\right)^{1/2},\quad v_{ks}=\frac{s}{\sqrt{2}}\left(1+\frac{\epsilon_k-\mu}{E_{ks}}\right)^{1/2} \quad (78)$$

The quantity $\xi_{ksjs'}$ is defined similarly to (73)

$$\xi_{ksjs'}=u_{ks}u_{js'}-v_{ks}v_{js'} \quad (79)$$

With the help of the quantities (77) and (79) the Fourier transform of (76) can be put into a form similar to the one at $\Delta=0$,

$$\chi_{FF}(\omega)=\frac{1}{2\hbar}\{\sum_{k=j}\sum_{s\neq s'}+\sum_{k\neq j}\sum_{ss'}\}\xi_{ksjs'}^2\frac{n_{ks}^T-n_{js'}^T}{\hbar\omega-(E_{ks}-E_{js'})+i\varepsilon}|F_{kj}|^2 \quad (80)$$

In the same way one may obtain the expressions for the other components of the intrinsic response tensor, see the Appendix.

3.2.3 Intrinsic response functions for collisional damping

To account for collisional widths of the quasi-particle states we proceed similarly to the no paring case [9, 8]. Let us explain details again at the example of the FF-response function. The essence of the procedure is most easily seen after first re-writing (80) in the form

$$\chi_{FF}(\omega)=-\frac{1}{2\pi\hbar}\{\sum_{k=j}\sum_{s\neq s'}+\sum_{k\neq j}\sum_{ss'}\}\xi_{ksjs'}^2|F_{kj}|^2\times$$
$$\int_{-\infty}^{\infty}d\Omega\, n^T(\Omega)\left[\varrho_{ks}(\Omega)\mathcal{G}_{js'}^{(0)}(\Omega-\omega-i\varepsilon)+\varrho_{js'}(\Omega)\mathcal{G}_{ks}^{(0)}(\Omega+\omega+i\varepsilon)\right] \quad (81)$$

where $\mathcal{G}_{ks}^{(0)}(\omega\pm i\varepsilon)$ is the Green function for independent quasi-particles

$$\mathcal{G}_{ks}^{(0)}(\omega\pm i\varepsilon)=\frac{1}{\hbar\omega-E_{ks}\pm i\varepsilon} \quad (82)$$

The spectral density ϱ_{ks} is related to $\mathcal{G}^{(0)}_{ks}(\omega \pm i\varepsilon)$ by

$$\varrho_{ks}(\omega) = i(\mathcal{G}^{(0)}_{ks}(\omega + i\varepsilon) - \mathcal{G}^{(0)}_{ks}(\omega - i\varepsilon)) \tag{83}$$

As argued in [22] the Green function $\mathcal{G}^{(0)}_{ks}(\omega \pm i\varepsilon)$ must be modified by introducing a spreading of the quasi-particle width. This may be achieved by introducing into the Green function a complex self-energy $\Sigma(\omega \pm i\varepsilon, \Delta, T) = \Sigma'(\omega, \Delta, T) \mp i\Gamma(\omega, \Delta, T)/2$

$$\mathcal{G}^{(0)}_{ks}(\omega \pm i\varepsilon) \to \mathcal{G}_{ks}(\omega \pm i\varepsilon) = \frac{1}{\hbar\omega - E_{ks} - \Sigma'(\omega) \pm i\Gamma(\omega)/2} \tag{84}$$

The way of how Σ is to be calculated in the presence of pairing will be discussed in the next subsection. With $\mathcal{G}_{ks}$ and ϱ_{ks} given the response function (81) takes the form

$$\chi_{FF}(\omega) = \{\sum_{k=j}\sum_{s\neq s'} + \sum_{k\neq j}\sum_{ss'}\}\xi^2_{ksjs'}\chi_{ksjs'}(\omega)|F_{kj}|^2 \tag{85}$$

where the amplitudes $\chi_{ksjs'}(\omega)$ are defined in the same way as for the system without pairing, see [8].

$$\chi_{ksjs'}(\omega) = \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi\hbar}\, n^T(\Omega)\, [\varrho_{ks}(\Omega)\mathcal{G}_{js'}(\Omega - \omega - i\varepsilon) + \varrho_{js'}(\Omega)\mathcal{G}_{ks}(\Omega + \omega + i\varepsilon)] \tag{86}$$

The only difference is that the energies $\epsilon_k - \mu$ or $\epsilon_j - \mu$ in eq.(4.2) of [8] are replaced by E_{ks} or $E_{js'}$.

The folding integral in (86) is computed by means of the residue theorem closing the integration contour in the lower half of the frequency plane. The residues which contribute to the integral (86) are that of $\mathcal{G}_{ks}(\Omega \pm i\varepsilon)$ (mind (83)) and $n^T(\Omega)$. In case of the frequency independent width $\Gamma = \Gamma(\Delta, \Delta, T)$ used here (see also sect.3.3 below) the poles of $\mathcal{G}_{ks}(\Omega \pm i\varepsilon)$ are given by

$$\hbar\Omega_{\pm} = E_{ks} \mp i\Gamma/2 \tag{87}$$

and the poles of $n^T(\Omega)$ are given by the Matsubara frequencies

$$\hbar\Omega_n = \pm i\pi T(2n+1), \qquad n = 0, 1, 2, ... \tag{88}$$

The computation of the folding integral (86) with a frequency dependent width $\Gamma(\omega, T)$ without pairing is discussed in detail in [8].

3.3 The quasi-particle strength function

The Green function $\mathcal{G}_{ks}$ (84) contains the frequency dependent self-energy Σ. Let us first focus our attention on its imaginary part $-Im\Sigma = \Gamma/2$, which we will estimate from the collision term of the Boltzmann equation for the scattering of two quasi-particles. Its standard form can be found in the review of Baym and Pethik [26] for ordinary particles and in [27, 28] for Bogolyubov quasi-particles. As is well known, for $\Delta = 0$ and for small excitations the width (decay rate) $\Gamma_d(\hbar\omega)$ can be written as

$$\Gamma_d(\hbar\omega) = \frac{1}{\Gamma_0}[(\hbar\omega - \mu)^2 + \pi^2 T^2]\,. \tag{89}$$

For Bogolyubov quasi-particles no analytic expression for this decay rate $\Gamma_d(\hbar\omega)$ is available, rather this quantity has to be computed numerically. In the present work we followed the publications [22, 29] where $\Gamma_d(\hbar\omega)$ was expressed as

$$\Gamma_d(\hbar\omega) = \frac{2}{\Gamma_0}\int\int\int d\epsilon_2 d\epsilon_3 d\epsilon_4 \delta(\hbar\omega+\mathcal{E}_2+\mathcal{E}_3+\mathcal{E}_4)[n_2^T n_3^T n_4^T + (1-n_2^T)(1-n_3^T)(1-n_4^T] \quad (90)$$

Here, the n_i^T represent the thermal equilibrium distribution $n^T(\mathcal{E}_i) = (1+\exp(\mathcal{E}_i/T))^{-1}$ for the quasi-particles, the energies of which are defined as

$$\mathcal{E}_k = E_k \frac{\epsilon_k - \mu}{|\epsilon_k - \mu|} \quad (91)$$

They are positive (negative) for states arising from single particle states above (below) the Fermi energy. The Γ_0 depends on the averaged matrix element of the residual interaction, the effective mass m^* at the Fermi energy and a cut-off momentum q_c:

$$\frac{1}{\Gamma_0} = \left(\frac{m^*}{2\pi\hbar}\right)^3 \frac{q_c}{p_F} \left|\overline{\langle 34|g|12\rangle}\right|^2 \quad (92)$$

In deriving (90) an expansion in the momenta around the Fermi momentum was used, and the q_c is an upper limit for the momentum transfer involved in the scattering, which is assumed to be small compared to the Fermi momentum $q_c(\leq p_F)$, see [26]. Therefore, the result (90) is valid for small excitations only. Fortunately, it is this region around the Fermi energy which gives the main contribution to the response functions. In case Γ_d is needed also for larger energies a correction becomes necessary. This problem is not specific to pairing, it already exists for the zero gap limit where it was solved by introducing one more parameter, see [22, 29]. Generalizing this procedure to the case of pair correlations, we may assume the final form of decay width to be given by

$$\Gamma = \frac{\Gamma_d}{1+\Gamma_d\Gamma_0/c^2} \quad (93)$$

with the cut-off parameter c (in energy) chosen to be independent of Δ and equal to the value $c = 20$ MeV given already in [22]. Likewise, for the Γ_0 of (92) we take the same constant as before, namely $\Gamma_0 = 33$ MeV (see [22]).

The computation of the folding integral (81) with the frequency dependent width Γ requires the knowledge of Γ in the whole range of ω from zero to infinity. This turns out to be a rather difficult task. Already a calculation of the Γ_d through (90)) would involve a double integral, which in the end would be too time consuming. To reduce the computation time to manageable level we chose to work with a constant Γ, taken at the Fermi energy. This is to say that in the present numerical evaluations we replaced $\Gamma(\hbar\omega)$ by $\Gamma = \Gamma(\hbar\omega = \Delta)$. This quantity is shown in Fig.2 as function of temperature for a few values of the pairing gap. As might be expected on general grounds, for $T = 0$ the collisional width (93) is zero for all $\hbar\omega \leq 2\Delta$ (incidentally, in [22] it was found to vanish in the regime $\hbar\omega \leq 3\Delta$). It is also seen that Γ is the smaller the larger is Δ. This effect is especially strong for small temperatures, say $T \leq 0.5$ MeV.

Once the $\Gamma(\hbar\omega, \Delta, T)$ is known the real part of Σ is given by Kramers-Kronig relation. For systems without pairing it was shown analytically [8] that $\Sigma' \sim \hbar\omega - \mu$. The same is true in the presence of pairing. The numerical results exhibit the Σ' to vanish at the Fermi

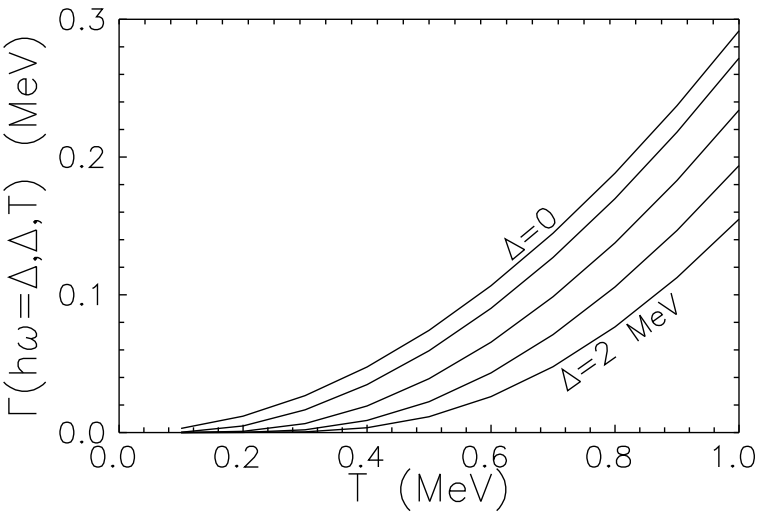

Figure 2: The collisional width (93) taken at the Fermi energy ($\hbar\omega = \Delta$) as function of the temperature. Different curves correspond to different values of pairing gap Δ indicated in the figure.

energy and to be very small in its vicinity. Within the approximations used here, namely to take the width at the Fermi energy where $\Sigma' = 0$, the real part of the self-energy does not contribute to the response functions.

Finally, we should like to mention some shortcomings of the derivation of (90) given in [22], which so far we have not been able to improve, but which for the present works. 1) In the vicinity of the critical temperature T_c, where the pairing gap is small, not only the scattering but also the coalescence and decay of quasi-particles should be taken into account, see [27]. 2) For small temperatures, where the pairing gap is large, the introduction of the energies $\mathcal{E}_k$, instead of quasi-particle energies E_k — which make the collision integral resemble that for ordinary particles as closely possible — does not appear to be well justified; moreover, one would think that the decay rate should also be influenced by the amplitudes u_k^2 and v_k^2. It is very likely, of course, that such finer details will not modify our final results too much, as we are using drastic approximations to the width anyway. More detailed investigations are required to clarify these points.

4 Average particle number conservation

In this chapter we address the problem of particle number conservation. As one knows, violation of any symmetry may lead to spurious states. Their existence in the present model may be seen with the help of the FF-response function, for instance. Its poles are associated with the energies of the systems' excited states. As it is seen from (75) some of the poles correspond to pair excited states $\alpha_k^\dagger \alpha_{\bar{k}}^\dagger |0\rangle$ with energies $2E_k$. The number of these states is equal to the number of terms in the sums over k or j in (75)), one more than the right value, as argued in [16]. This extra states is the spurious one. Fortunately, it is possible to construct response functions where such contributions are removed, as shall be explained now.

4.1 Number conserved response

4.1.1 Modified intrinsic response

For the independent quasi-particle model applied to the quasi-static picture, it is the chemical potential μ which guarantees the conservation of the average particle number determined by eq.(46). However, the dynamical density matrix will differ from the quasi-static density $\rho_{\rm qs}$. Therefore, in a time dependent picture the average particle number $\langle N\rangle_t$ does not automatically coincide with N. Consequently, spurious contributions proportional to $\delta\langle\hat{N}\rangle_t$ might appear. Following [17]and [18], one may correct this drawback *requiring as a constraint* $\delta\langle\hat{N}\rangle_t = 0$ to hold true, thus warranting that the average particle number is conserved also in dynamical sense.

To implement such a constraint into the theory let us recall first that the Hamiltonian (38) depends on the shape parameter Q, the chemical potential μ and Δ. The change with time of the average of some operator, say $\langle\hat{F}\rangle_t$, will generally depend on the variation of all three parameters Q, μ and Δ. To evaluate the deviation of $\langle\hat{F}\rangle_t$ one may use the set of equations (12) which in the present case read:

$$\delta\langle\hat{F}\rangle_\omega = -\chi_{FF}(\omega)\delta Q(\omega) + \chi_{FN}(\omega)\delta\mu(\omega) + \chi_{FP}(\omega)\delta\Delta(\omega) \tag{94}$$

$$\delta\langle(\hat{P}+\hat{P}^\dagger)\rangle_\omega = -\chi_{PF}(\omega)\delta Q(\omega) + \chi_{PN}(\omega)\delta\mu(\omega) + \chi_{PP}(\omega)\delta\Delta(\omega) \tag{95}$$

A similar relation must hold true for $\delta\langle\hat{N}\rangle_\omega$

$$\delta\langle\hat{N}\rangle_\omega = -\chi_{NF}(\omega)\delta Q(\omega) + \chi_{NN}(\omega)\delta\mu(\omega) + \chi_{NP}(\omega)\delta\Delta(\omega) \tag{96}$$

As mentioned previously, the expressions for the FN-, NP-response functions may be found in the Appendix. The condition $\delta\langle\hat{N}\rangle_t = 0$ or $\delta\langle\hat{N}\rangle_\omega = 0$ together with (96) allows one to express $\delta\mu(\omega)$ in terms of $\delta Q(\omega)$ and $\delta\Delta(\omega)$

$$\delta\mu(\omega) = \frac{\chi_{NF}(\omega)}{\chi_{NN}(\omega)}\delta Q(\omega) - \frac{\chi_{NP}(\omega)}{\chi_{NN}(\omega)}\delta\Delta(\omega) \tag{97}$$

By substituting (97) into (94) and (95) the latter two equations turn into

$$\begin{aligned}\delta\langle\hat{F}\rangle_\omega &= -\widehat{\chi}_{FF}(\omega)\delta Q(\omega) + \widehat{\chi}_{FP}(\omega)\delta\Delta(\omega)\\ \delta\langle(\hat{P}+\hat{P}^\dagger)\rangle_\omega &= -\widehat{\chi}_{PF}(\omega)\delta Q(\omega) + \widehat{\chi}_{PP}(\omega)\delta\Delta(\omega)\end{aligned} \tag{98}$$

In this way we get new response functions $\widehat{\chi}_{FF}(\omega)$, $\widehat{\chi}_{FP}(\omega)$ and $\widehat{\chi}_{PP}(\omega)$ which account for the conservation of the average particle number during a dynamical process. They may be constructed from the former one by the simple relations:

$$\widehat{\chi}_{FF}(\omega) = \chi_{FF}(\omega) - \frac{\chi_{FN}(\omega)\chi_{NF}(\omega)}{\chi_{NN}(\omega)} \tag{99}$$

$$\widehat{\chi}_{FP}(\omega) = \chi_{FP}(\omega) - \frac{\chi_{FN}(\omega)\chi_{NP}(\omega)}{\chi_{NN}(\omega)} \tag{100}$$

$$\widehat{\chi}_{PP}(\omega) = \chi_{PP}(\omega) - \frac{\chi_{PN}(\omega)\chi_{NP}(\omega)}{\chi_{NN}(\omega)} \tag{101}$$

4.1.2 Modified collective response

We may now use the information contained in (99)-(101), (63), and (60) to construct the collective response tensor (18). The expressions for the component of $\boldsymbol{\chi}_{\rm coll}(\omega)$ can be obtained directly from the form given in (18). One gets

$$\chi^{\rm coll}_{FF}(\omega) = \frac{\kappa_Q[(\kappa_\Delta + \hat{\chi}_{PP})\hat{\chi}_{FF} - \hat{\chi}_{FP}\hat{\chi}_{FP}]}{(\kappa_Q + \hat{\chi}_{FF})(\kappa_\Delta + \hat{\chi}_{PP}) - \hat{\chi}_{FP}\hat{\chi}_{PF}} \tag{102}$$

$$\chi^{\rm coll}_{FP}(\omega) = \frac{\kappa_Q\kappa_\Delta\hat{\chi}_{FP}}{(\kappa_Q + \hat{\chi}_{FF})(\kappa_\Delta + \hat{\chi}_{PP}) - \hat{\chi}_{FP}\hat{\chi}_{PF}} \tag{103}$$

and

$$\chi^{\rm coll}_{PP}(\omega) = \frac{\kappa_\Delta[(\kappa_Q + \hat{\chi}_{FF})\hat{\chi}_{PP} - \hat{\chi}_{FP}\hat{\chi}_{FP}]}{(\kappa_Q + \hat{\chi}_{FF})(\kappa_\Delta + \hat{\chi}_{PP}) - \hat{\chi}_{FP}\hat{\chi}_{PF}} \tag{104}$$

The collective Q-vibrational frequency (Ω_1) is defined by the lowest pole of collective response function (102) or by the equation

$$(\kappa_Q + \hat{\chi}_{FF}(\Omega_1))(\kappa_\Delta + \hat{\chi}_{PP}(\Omega_1)) - \hat{\chi}_{FP}(\Omega_1)\hat{\chi}_{PF}(\Omega_1) = 0. \tag{105}$$

It is not difficult to convince oneself that (105) may as well be interpreted as the secular equation of the system (94)-(96), provided one obeys the subsidiary condition $\delta\langle\hat{N}\rangle_\omega = 0$ as well as the Q- and Δ-components of self-consistency conditions (13). In case we are interested in vibrations around a minimum of the potential energy, both with respect to Q and Δ, we may identify Q_m with Q_0, Δ_m with Δ_0. Thus the Q- and Δ-components of $q(\omega)$ will coincide with $\delta Q(\omega)$ or $\delta\Delta(\omega)$ and (13) will attain the form

$$\delta\langle\hat{F}\rangle_\omega = \kappa_Q\delta Q(\omega)\,, \quad \delta\langle\hat{P} + \hat{P}^\dagger\rangle_\omega = \frac{2}{G}\delta\Delta(\omega) \tag{106}$$

Excluding $\delta\langle\hat{F}\rangle_\omega$, $\delta\langle\hat{P} + \hat{P}^\dagger\rangle_\omega$ and $\delta\langle\hat{N}\rangle_\omega$ from the system (94)-(96) with the help of (106) together with $\delta\langle\hat{N}\rangle_\omega = 0$ one will get the following system of equations

$$\begin{array}{rrrr} -(\chi_{FF}(\omega) + \kappa_Q)\delta Q(\omega) & +\chi_{FN}(\omega)\delta\mu(\omega) & +\chi_{FP}(\omega)\delta\Delta(\omega) = 0 \\ -\chi_{NF}(\omega)\delta Q(\omega) & +\chi_{NN}(\omega)\delta\mu(\omega) & +\chi_{NP}(\omega)\delta\Delta(\omega) = 0 \\ -\chi_{PF}(\omega)\delta Q(\omega) & +\chi_{PN}(\omega)\delta\mu(\omega) & +(\chi_{PP}(\omega) - 2/G)\delta\Delta(\omega) = 0 \end{array} \tag{107}$$

The secular equations is then given by putting the determinant of the matrix behind this set equal to zero. Indeed, this leads to the same result as in (105). Below, this fact shall be exploited further for special cases.

4.2 RPA at zero thermal excitations

It should be worth while to compare our results with those obtained in previous treatments at zero temperature. Using the expressions for the intrinsic response functions given in the Appendix for $T = 0$ one derives from (107) the following secular equation

$$\mathrm{Det}_{LRT}(\omega) \equiv \begin{vmatrix} \sum_{kj}\frac{2\eta^2_{kj}E_{kj}|F_{kj}|^2}{E^2_{kj}-\hbar^2\omega^2} + \kappa_Q & 4\Delta\sum_k\frac{\eta_{kk}F_{kk}}{4E^2_k-\hbar^2\omega^2} & 4\Delta\sum_k\frac{\xi_{kk}F_{kk}}{4E^2_k-\hbar^2\omega^2} \\ 4\Delta\sum_k\frac{\eta_{kk}F_{kk}}{4E^2_k-\hbar^2\omega^2} & 4\Delta\sum_k\frac{\eta_{kk}}{4E^2_k-\hbar^2\omega^2} & 4\Delta\sum_k\frac{\xi_{kk}}{4E^2_k-\hbar^2\omega^2} \\ 4\Delta\sum_k\frac{\xi_{kk}F_{kk}}{4E^2_k-\hbar^2\omega^2} & 4\Delta\sum_k\frac{\xi_{kk}}{4E^2_k-\hbar^2\omega^2} & \sum_k\frac{4E_k\xi^2_{kk}}{4E^2_k-\hbar^2\omega^2} - \frac{2}{G} \end{vmatrix} = 0 \tag{108}$$

This equation may be compared with the secular equations obtained in [19, 20] and used for investigations of vibrational states in medium and heavy nuclei. The determinants of [19] and [20] differ from each other by a constant multiplier so that the collective frequencies are the same. The one obtained in [20] (let us denote it as $\mathrm{Det}_{RPA}(\omega)$) writes

$$\mathrm{Det}_{RPA}(\omega) \equiv \begin{vmatrix} \sum_{kj} \frac{|F_{kj}|^2 2\eta_{kj}^2 (E_k+E_j)}{(E_k+E_j)^2-\omega^2} + \frac{1}{k} & \omega \sum_k \frac{\eta_{kk} F_{kk}}{4E_k^2-\omega^2} & 2\sum_k \frac{\xi_{kk}\eta_{kk} F_{kk}}{4E_k^2-\omega^2} \\ 2\omega \sum_k \frac{\eta_{kk} F_{kk}}{4E_k^2-\omega^2} & \sum_k \frac{2E_k}{4E_k^2-\omega^2} - \frac{1}{G} & \omega \sum_k \frac{\xi_{kk}}{4E_k^2-\omega^2} \\ 4\sum_k \frac{\xi_{kk}\eta_{kk} F_{kk}}{4E_k^2-\omega^2} & \omega \sum_k \frac{\xi_{kk}}{4E_k^2-\omega^2} & \sum_k \frac{2E_k \xi_{kk}^2}{4E_k^2-\omega^2} - \frac{1}{G} \end{vmatrix} \tag{109}$$

To establish a relation between (108) and (109) let us first recall that in [19, 20] vibrations around the minimum of the potential energy were considered. In this case, because of the linearization involved, the pairing gap may be defined through the solution of the gap equation (corresponding to the minimum) and one may replace $1/G$ by $\sum_k 1/2E_k$. Consequently, one has

$$\sum_k \frac{2E_k}{4E_k^2-\omega^2} - \frac{1}{G} = \frac{\omega^2}{2\Delta} \sum_k \frac{\eta_{kk}}{4E_k^2 - \hbar^2\omega^2} \tag{110}$$

Considering this relation, the determinants (108) and (109) look very similar. Indeed, by explicit calculation one may verify the following relation

$$\mathrm{Det}_{RPA}(\omega) = \frac{\omega^2}{(4\Delta)^2} \mathrm{Det}_{LRT}(\omega) \tag{111}$$

It is clear from (111) that our secular equation (108) defines the nontrivial part of the secular equation of [19, 20]. The spurious mode corresponding to $\omega = 0$ is not present among the solutions of (108), which are the same as those found in [19, 20].

Later on we are going to evaluate transport coefficients. They are largely determined by the strength distributions associated to the frequency dependence of response functions, like $\chi_{FF}(\omega)$ and $\hat{\chi}_{FF}(\omega)$. Our numerical computations show that the shift in the strength distribution due to correction terms like $\chi_{FN}(\omega)\chi_{FN}(\omega)/\chi_{NN}(\omega)$ is small. This implies that particle number conservation has only little influence on the transport coefficients. The correction terms mostly cause shifts in the positions of the peaks of (the imaginary parts of) the response function, whereas the widths of the latter remain almost unchanged. As a consequence, the friction coefficients deduced from $\chi_{FF}(\omega)$ and $\hat{\chi}_{FF}(\omega)$ are almost identical. Not much difference is seen is also in the mass parameter and the local stiffness.

4.2.1 Pairing without shape oscillations

For a comparison with the standard pairing theory let us consider the particular case where the coupling between Q-mode and Δ-mode may be neglected ($\hat{\chi}_{FP}(\omega) = 0$). In this case the secular equation (105) separates into individual ones for the Q- and the Δ-mode. The latter can be written as

$$\widehat{\chi}_{PP}(\omega) - \frac{2}{G} = \chi_{PP}(\omega) - \frac{2}{G} - \frac{\chi_{PN}(\omega)\chi_{NP}(\omega)}{\chi_{NN}(\omega)} = 0 \tag{112}$$

Replacing $2/G$ in the same way as above by $\sum_k 1/E_k$ and using the explicit expressions for the response functions, eq.(112) is transformed to

$$(\hbar^2\omega^2 - 4\Delta^2)\left(\sum_k \frac{1}{E_k(4E_k^2 - \hbar^2\omega^2)}\right)^2 = \left(\sum_k \frac{2(\epsilon_k - \mu)}{E_k(4E_k^2 - \hbar^2\omega^2)}\right)^2 \tag{113}$$

After taking the square root eq.(113) is reduced to

$$\sum_k \frac{\sqrt{\hbar^2\omega^2 - 4\Delta^2} \pm 2(\epsilon_k - \mu)}{E_k(4E_k^2 - \hbar^2\omega^2)} = 0 \tag{114}$$

This equation is identical to that obtained in [16, 21] within quasi-boson approximation. In this sense essential results of the theory of vibrational states in paired systems are reproduced by our linear response approach.

5 Numerical results

In this section we are going to present numerical results for response functions and transport coefficients. We concentrate on deformations along the fission path of the nucleus ^{224}Th, which has been under investigation before. Indeed, present one is an extension of previous publications [30, 8] where temperatures above $T = 1MeV$ had been considered with pairing discarded. Like in [8] we use the independent particle Hamiltonian based on the deformed Woods-Saxon potential with the nuclear shape parameterized in terms of Cassini ovaloids [31]. The Cassini ovaloids are defined by rotating the curve

$$\rho(z, \epsilon) = R_0\left[\sqrt{a^4 + 4\epsilon z^2/R_0^2} - z^2/R_0^2 - \epsilon\right]^{1/2} \tag{115}$$

around the z-axis, with z and ρ being cylindrical coordinates. The constant a is determined from volume conservation, implying that the family of shapes (115) depends only on one deformation parameter ϵ. As is easily recognized from (115) the value of $\epsilon = 0$ corresponds to a sphere. For $0 < \epsilon < 0.4$ the form (115) resembles very much that of a spheroid with the ratio of the axes given by

$$\frac{\text{shorter axes}}{\text{longer axes}} = \frac{1 - 2\epsilon/3}{1 + \epsilon/3} \tag{116}$$

At $\epsilon \approx 0.5$ a neck appears and at $\epsilon = 1.0$ the nucleus separates into two fragments. Like in [8], instead of ϵ we will use the parameter $Q = R_{12}/2R_0$, which measures the distance R_{12} between the left and right center of masses divided by the diameter $2R_0$ of the sphere of identical volume. Besides having a simple physical meaning, the $Q = R_{12}/2R_0$ allows one to relate to other shape parameterizations of deformed potentials.

In principle, the pairing gap is different for neutrons and protons. Thus, for quantitative studies one should introduce gaps for protons and neutrons as two independent dynamical variables. However, in the present work we would like to concentrate more on questions of principle nature, as a general investigating of the effects of pairing on transport coefficients. Thus we feel justified to compute the response functions putting $\Delta^p = \Delta^n = \Delta$ and to use this Δ as a collective parameter common for protons and neutrons. This will simplify also the presentation of our results.

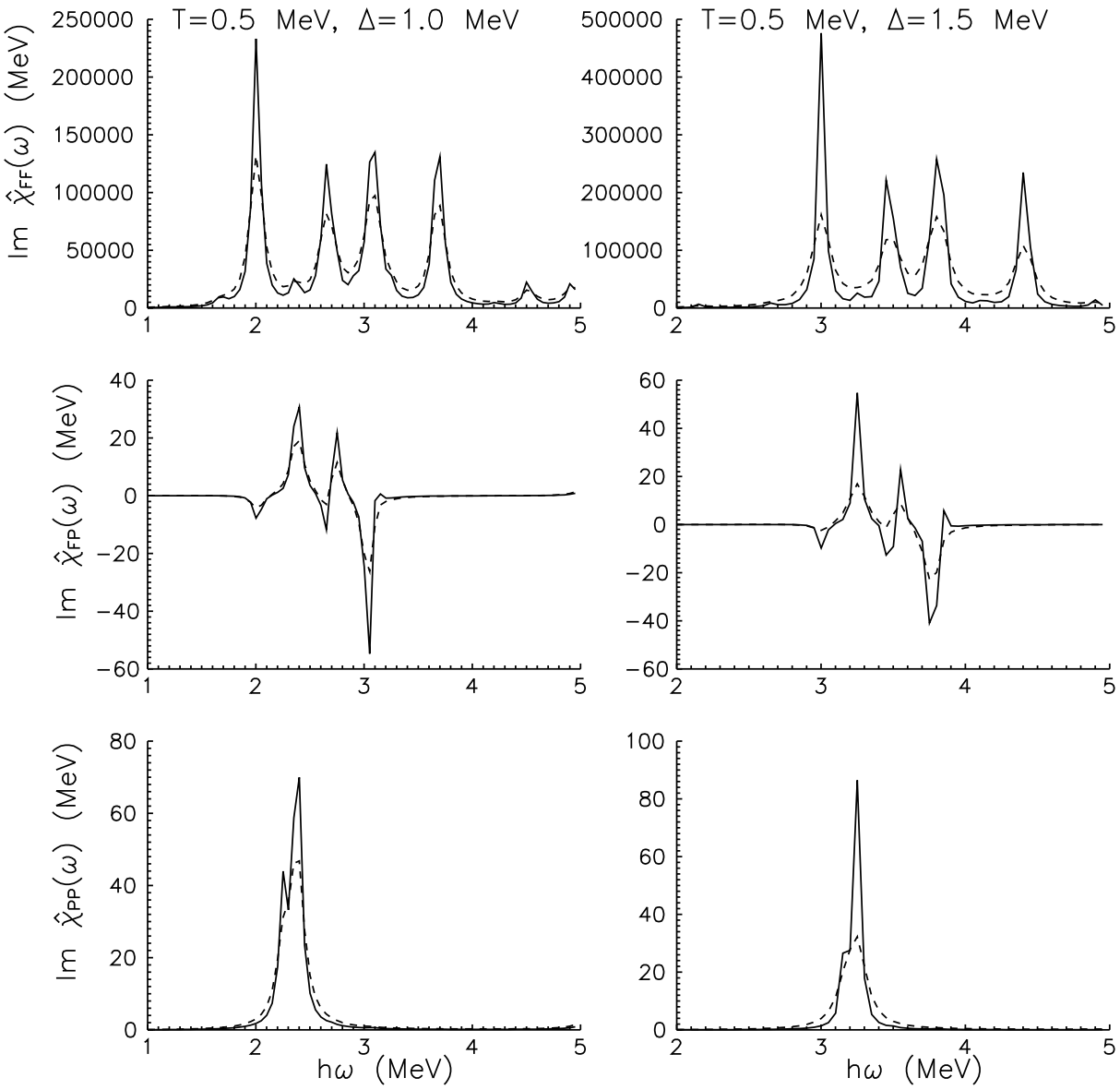

Figure 3: The frequency dependence of the imaginary part of the intrinsic response functions (99)-(101). The computations were done with the spherical Woods-Saxon potential for the nucleus ^{224}Th. The values of the temperature T and Δ are indicated in the figure. The dashed and solid curves correspond to different choices of the collisional widths, namely (89) and (90), respectively (together with (93)).

5.1 Information from the intrinsic response

We begin looking at the intrinsic response functions $\chi_{\mu\nu}$, as the most basic quantity from which all local transport properties may be deduced, once the static energy is known. For instance, it allows for an immediate calculation of inertia and friction within the zero frequency limit, which for not too small temperature may give reasonably accurate results, in particular in the version suggested in [8]. Moreover, this response function is a basic element in the construction of the collective response function, which in turn allows one to evaluate the implication of self-consistency on transport coefficients. It is thus of great interest to look first at the effects pairing has on the $\chi_{\mu\nu}$. In Fig.3 we show the dissipative parts of the susceptibilities given in (99)-(101) as function of frequency, for $T = 0.5MeV$ and two values of Δ, for spherical configurations. The functions presented in all three parts of the figure exhibit a narrow peak structure. These peaks represent the excitations the system would have if there were no correlations due to collective motion.

Let us first look at the effects the collisional width $\Gamma(\hbar\omega = \Delta, \Delta, T)$ has on width and height of the peaks in the response functions. To clarify this influence, and consequently the one on the transport coefficients, in Fig.3 two sets of curves are plotted: One computed with $\Gamma(\hbar\omega = \Delta, \Delta = 0, T)$ (dashed curves) and another one with the $\Gamma(\hbar\omega = \Delta, \Delta, T)$ given by eqs.(93) and (90). As it is seen from the figure, the peaks computed with a finite Δ are higher and more narrow as compared with those for the case $\Delta = 0$. This is to be expected since, as it is seen from Fig.2 for fixed temperature, $\Gamma(\hbar\omega = \Delta, \Delta, T)$ is the smaller the larger is Δ. For fixed Δ the width $\Gamma(\Delta, \Delta, T)$ increases with temperature and, as the computations demonstrate, the response functions become smoother.

Next we examine the influence of the spectrum of the individual excitations. As compared to the unpaired case, the response functions shown in Fig.3 exhibit a totally different structure: The lowest peak of considerable strength occurs at or above the frequency $\hbar\omega \approx 2\Delta$, and the strength in the region $\hbar\omega \leq 2\Delta$ is very small. In case of the PP-response, for instance, the position of the peaks is defined by the zeros of eq.(114). The two peaks seen at the bottom part of Fiq.3 represent the contributions from the lowest solutions of eq.(114) for neutrons and protons. For a spherically symmetric potential the lowest peaks are well isolated from the rest. This is because the positions of the peaks of the PP-response function is mainly determined by the energy $2E_k$ of pair excitations. This distance is especially large for spherical configurations, simply because of the large degeneracies of the single-particle states. For deformed shapes the distance to the next peaks becomes much smaller.

Obviously, these features will have great impact on transport coefficients. This is most apparent for the friction coefficients defined in zero frequency limit: They will be the smaller the larger is Δ. This property also shows up more or less strongly for the self-consistent calculation. Let us therefore analyze the structure of the response functions at small excitations in more detail. It is easily recognized that it may be traced back to the various contributions to formula (75) (or the corresponding one resulting after collisions are taken into account). As we will see, the contributions from the two lines behave very differently, depending very sensitively on temperature. Let first look at the case of finite temperatures. Here, the first line of (75), resulting from the terms with $|E_k - E_j| \leq 2\Delta$, will have finite contributions even in the region $\hbar\omega \leq 2\Delta$. However, on the scale used in the figure, they are very small. Fig.4 shows the (dissipative part of the) response function in the interval below 2Δ in more detail, exhibiting clearly the existence of such

contributions.

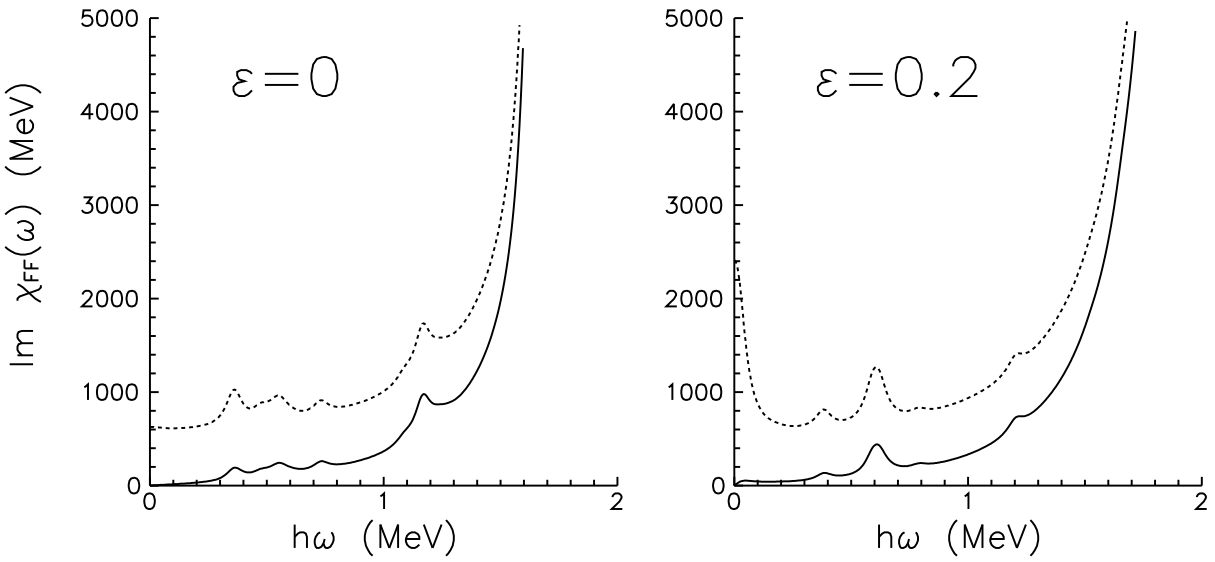

Figure 4: The frequency dependence of the imaginary parts of the response (solid curves) and correlation (dotted curves) functions. The computations are done with the spherical (left) and deformed (right, $\epsilon = 0.2$) potentials.

Since around $\omega = 0$, the region which is of special interest, this dissipative part has to vanish exactly, it is more instructive to use the *correlation function* $\psi''_{\mu\nu}(\omega)$, related to the former by the fluctuation dissipation theorem (for the nucleonic degrees of freedom), see [9]. One has to replace the commutator of (11) by an anti-commutator and must account for the unperturbed averages relative to which the deviations $\delta F_\mu = F_\mu - \langle F_\mu \rangle$ of the operators are to be defined,

$$\widetilde{\psi}''_{\mu\nu}(t) = \frac{1}{2}(\langle [\hat{F}^I_\mu(t), \hat{F}_\nu]_+ \rangle - \langle \hat{F}_\mu \rangle \langle \hat{F}_\nu \rangle) \tag{117}$$

Calculating the averages in (117) in the same way as for the response functions, for independent quasi-particles the Fourier transform of the FF-correlation function turns out to be

$$\begin{aligned} \psi_{FF}(\omega) = &-{\sum_{kj}}' n^T_k(1-n^T_j)\xi^2_{kj}\left(\frac{1}{\hbar\omega - E^-_{kj} + i\varepsilon} + \frac{1}{\hbar\omega + E^-_{kj} + i\varepsilon}\right)|\delta F_{kj}|^2 \\ &-\sum_{kj}[(n^T_k n^T_j + (1-n^T_k)(1-n^T_j)]\eta^2_{kj}\left(\frac{1}{\hbar\omega - E^+_{kj} + i\varepsilon} + \frac{1}{\hbar\omega + E^+_{kj} + i\varepsilon}\right)|\delta F_{kj}|^2 \end{aligned} \tag{118}$$

with $E^\pm_{kj} \equiv E_k \pm E_j$. The account of collisional damping is carried out in full analogy to the response function. The $\psi''_{FF}(\omega)$ is shown in Fig.4 by the dotted line. Comparing with the response function (full line) the properties just mentioned are clearly visible. In addition, another important and generic feature is revealed. At certain deformations the correlation function not only is finite at and around $\omega = 0$ but shows a more or less pronounced and sharp peak. For the special case shown here, which corresponds to a deformation of $\epsilon = 0.2$, it appears to lie exactly at $\omega = 0$. In this sense it reminds one of the "heat pole", which in previous publications was seen to become more relevant

at larger temperatures, see [32] and [9]. However, there are cases where such a peak is shifted to small but finite frequencies. These peaks appear whenever a pair of levels close to the Fermi energy approach each other; in the present case their distance is about $|\epsilon_k - \epsilon_j| \approx 0.02$ MeV which is definitely smaller than the width associated with such excitations.

Having the correlation function at one's disposal, the friction coefficient in zero frequency limit can be expressed as

$$\gamma_{FF}(0) = \psi''_{FF}(\omega = 0)/(2T) \tag{119}$$

Hence, for this limit it becomes apparent how peaks in the correlation function at $\omega = 0$ are related to peaks in the deformation dependence of the friction coefficient. For the special example discussed here it may be said that the above mentioned peak in ψ''_{FF} at $\omega = 0$ results in a peak of the friction coefficient shown in Fig.5, namely at $R_{12}/2R_0 \approx 0.43$ which corresponds to $\epsilon = 0.2$.

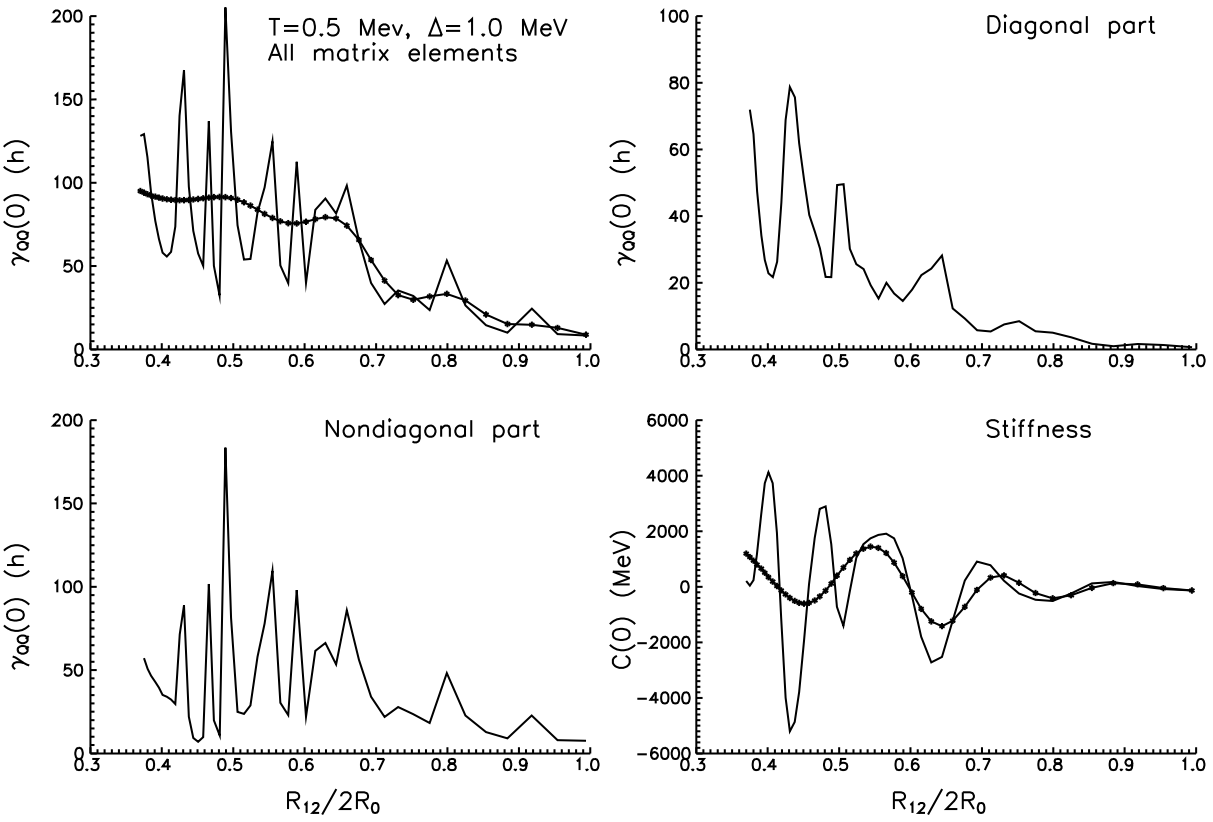

Figure 5: The zero frequency limit (25) for the friction coefficient in the shape degree of freedom Q as function of Q for $T = 0.5$ MeV and $\Delta = 1.0$ MeV. The right bottom and top left parts of figures show separately the contribution to the friction of nondiagonal F_{kj} and diagonal F_{kk} matrix elements. The bottom right part contains the stiffness of the static energy at $T = 0.5$ MeV. The curves with stars mark the value obtained by averaging over deformation on the interval $\Delta(R_{12}/2R_0) = 0.08$.

Let us turn to the extreme case of $T = 0$ for which Δ will be finite and, more important, for which the collisional width will be zero at least for $\hbar\omega$ up to $\hbar\omega = 2\Delta$; recall the discussion below eq.(93). Look at the forms (75) and (118) the response and correlation

functions attain for the case of no collisions. Then, only contributions from the excitations above 2Δ survive. In case of the presence of collisions, in the region $\omega \leq 2\Delta$ both the response and correlation functions may differ from zero only if tails from distant peaks extend into this regime. This is very unlikely to happen if the convolution integral in (86) is calculated exactly with the proper frequency dependent widths. Hence, friction in zero frequency limit will be strictly equal to zero. However, in practical computations where the convolution integrals in (86) are computed with constant Γ's such tails might be present, which in turn may lead to a finite though small value of the friction coefficient. The choice used in our calculations, namely $\Gamma = \Gamma(\Delta, \Delta, T)$, as described below (86), does not suffer from this drawback.

As the zero frequency limit provides the simplest definition of friction and inertia (see (25) and (26)) respectively, we like to stick to this version for a moment. In Fig.5 we exhibit the deformation dependence of friction for the Q-mode, as evaluated from (25). In this figure the rapid oscillations with deformation we spoke of can be seen. The inertia looks similar to friction. The formulas of the zero frequency limit (25)-(26) are simple enough to allow one separating the contributions from diagonal and non-diagonal parts of the response function (75) (diagonal here means the contribution with $k = j$). The source of fluctuations of these two parts turns out to be different. The diagonal sum (contribution of the diagonal part of the second sum in (75)) is affected strongly by the density of states near the Fermi energy, which is typical of the appearance of shell effects, of such a type as they also appear in the static energy. They exist as long as the temperature does not exceed a typical value of the order of $1 - 2$ MeV. The stiffness $C(0)$ of the static energy is shown in the right bottom part of Fig.5. Indeed, the same kind of fluctuations appear, only anti-correlated to the diagonal component of friction. The fluctuations of the non-diagonal part are caused by the quasi-crossings of levels near the Fermi energy like those discussed in connection to Fig.4. For each sharp peak in the left-hand-side of Fig.5 one can find in the single-particle spectrum one or a few quasi-crossings which contribute 90% or so to friction or inertia. The height of such peaks is the larger the smaller the minimal distance is between quasi-crossing levels. From the analysis of the single-particle spectrum computed with the deformed Woods-Saxon potential it follows that some of these minimal distances are extremely small - of the order of $0.02 - 0.03 MeV$.

For several reasons we claim that to large extent such closely spaced levels are shortcomings of the underlying shell model, and that they may lead to unphysical consequences. First of all, it should be noted that they are intimately related to the choice of the shape parameter, in particular to the restriction to only a few of them, more precisely to exactly one in the present case. In a truly multi-dimensional landscape such quasi-crossings would happen much less frequently as there is always enough room for avoiding them. Secondly, residual interactions would reduce their significance further. Indeed, from the statistics of experimentally measured nuclear states it is known that levels with small spacings are very seldom; the distribution of nearest neighbor spacings is of Wigner type rather than of Poisson type. Such interactions would repel the approaching states and in this way would reduce the matrix elements with the generator $\hat{F}$ of collective motion. Evidently, it should be the aim of any microscopic theory to deal with each of such problems individually. At present, we are not yet able to do so. Rather, we suggest to overcome these difficulties in another way, namely by averaging the transport coefficients over deformation. The averaging interval should be large enough to smooth out the rapid oscillations, as well as the finer details of the shell structure, but at the same time small enough to preserve

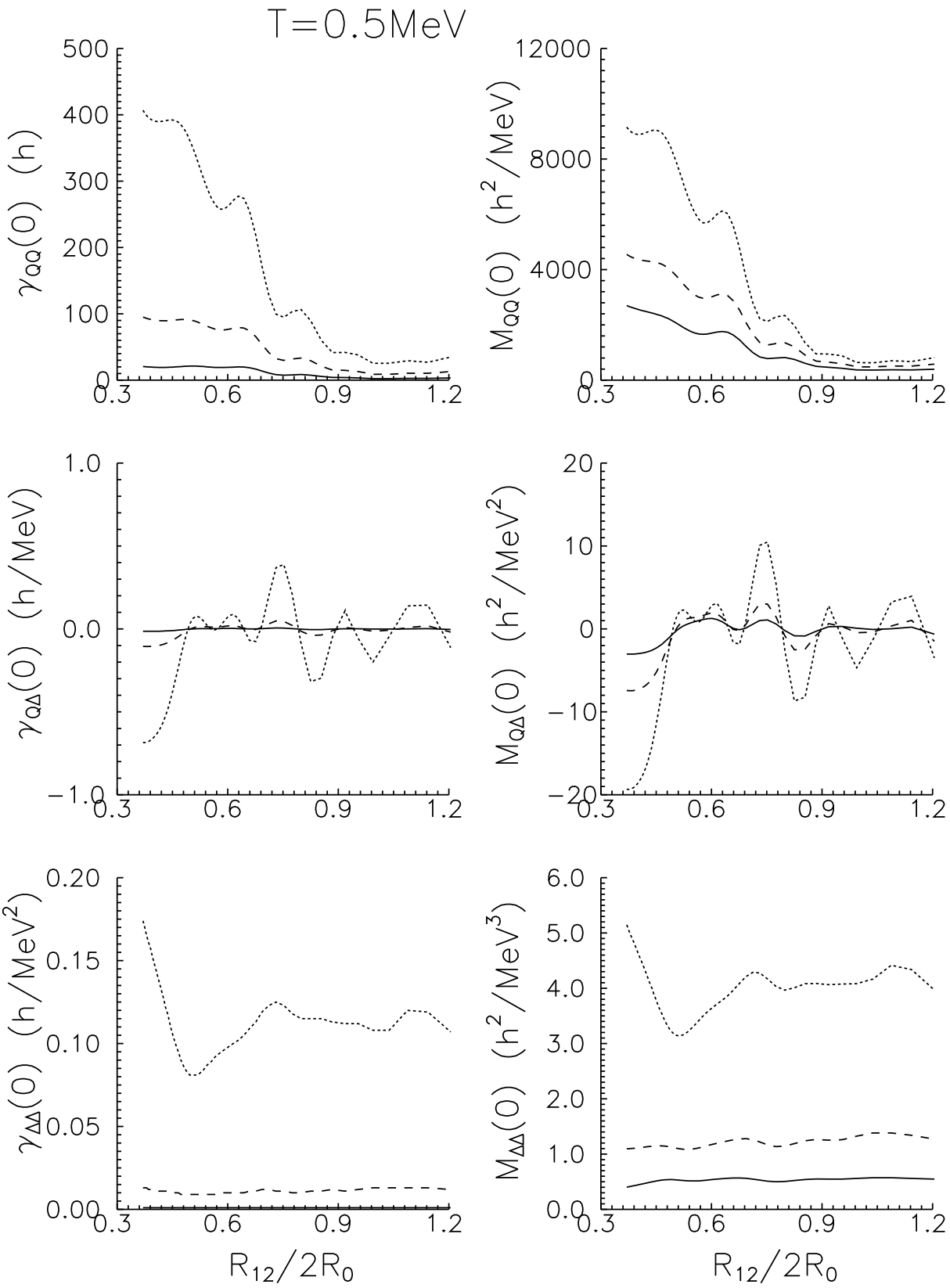

Figure 6: The averaged values of the components of the friction (left-hand-side) and inertial (right-hand-side) tensors obtained within zero frequency limit (25),(26) as function of deformation for temperature $T = 0.5$ MeV. The dotted, dashed and solid curves correspond to $\Delta = 0.5, 1.0$ and 1.5 MeV.

gross shell effects. This procedure may perhaps be motivated further by recalling the following facts. (i) Evidently, any structure in the transport coefficients of greater detail cannot be seen experimentally. Usually, one is happy if one succeeds to identify gross shell behavior. (ii) Finally, these transport coefficients are to be used in a transport equation of Fokker-Planck or Langevin type which account for genuine fluctuations in the collective variable. Without any doubt, such fluctuations will smooth out the detailed structure in the coefficients in most natural way. Incidentally, we may phrase our problem in a different way, namely in attributing it to a deficiency of the mean field approximation, at least partly.

The averaged friction coefficient and stiffness are shown by the curves with stars in Fig.5. As it is seen the frequent oscillations are gone while the typical gross structure remains. This latter feature is clearly visible at the case of the stiffness which now shows the typical behavior expected for a potential with a second (and perhaps third) minimum.

In Fig.6 we exhibit the averaged tensors of friction and inertia are shown, for $T = 0.5$ MeV and for a few values of the pairing gap Δ. Both friction and inertia still oscillate as function of the deformation, but in much weaker fashion than those shown in Fig.5. Now they correlate with the fluctuations of the stiffness or the shell correction. The non-diagonal terms, namely $Q\Delta$-friction and inertia, oscillate around an average value which is very small. Very likely this non-diagonal components of friction or inertia may simply be neglected. The $\Delta\Delta$-friction or inertia fluctuate much less as compared with the corresponding quantities in Q. The reason is that the matrix elements of $\hat{F}$ (which are of strongly peaked structure) do not contribute to $\Delta\Delta$-friction or inertia, see (101). The quantities which matter here most are the single-particle energies themselves which are relatively smooth functions of deformation. As seen from Fig.6, an increase of Δ has two effects, both on friction as well as on inertia: They become smaller and their Q-dependence becomes smoother with growing Δ. In case of the QQ-transport coefficients the substantial contribution comes from the diagonal component of the $\hat{F}$ operator (diagonal term in eq.(74)). For not too small Δ his diagonal contribution can be estimated to behave like $\propto 1/\Delta^2$ both for friction and inertia. The temperature dependence (for fixed Δ) of friction and inertia is the same as in the no pairing case: Whereas friction increases with temperature, inertia decreases.

5.2 Information from the collective response

In Fig.7 a few examples of the tensor of the collective response are shown for some values of Δ and temperature. A common feature of all figures is as follows: The collective frequency is shifted to smaller frequencies as compared to the lowest peak of the intrinsic response. The magnitude of the shift depends on the value of the static response $\chi'_{\mu\nu}(0)$ relative to the stiffness $C_{\mu\nu}(0)$, see Fig.6 of [12] for a demonstration. The smaller is the stiffness the larger is the shift of collective strength to lower frequencies. In case of the Q-mode the static stiffness is much smaller than $\chi'_{FF}(0)$ and, hence, the collective strength concentrates at low frequencies. The situation turns out very different for the Δ-mode, whose stiffness $C_{\Delta\Delta}$ approximately equals the static response $\chi'_{PP}(0)$ implying that the shift of the collective peak is small.

In ideal cases the $\chi^{\rm coll}_{\mu\nu}(\omega)$ is dominated by one peak in the low frequency region. This peak may then be fitted rather accurately by the response function of the damped oscillator, in which way the self-consistent transport coefficients are defined, see [8]. Unfortu-

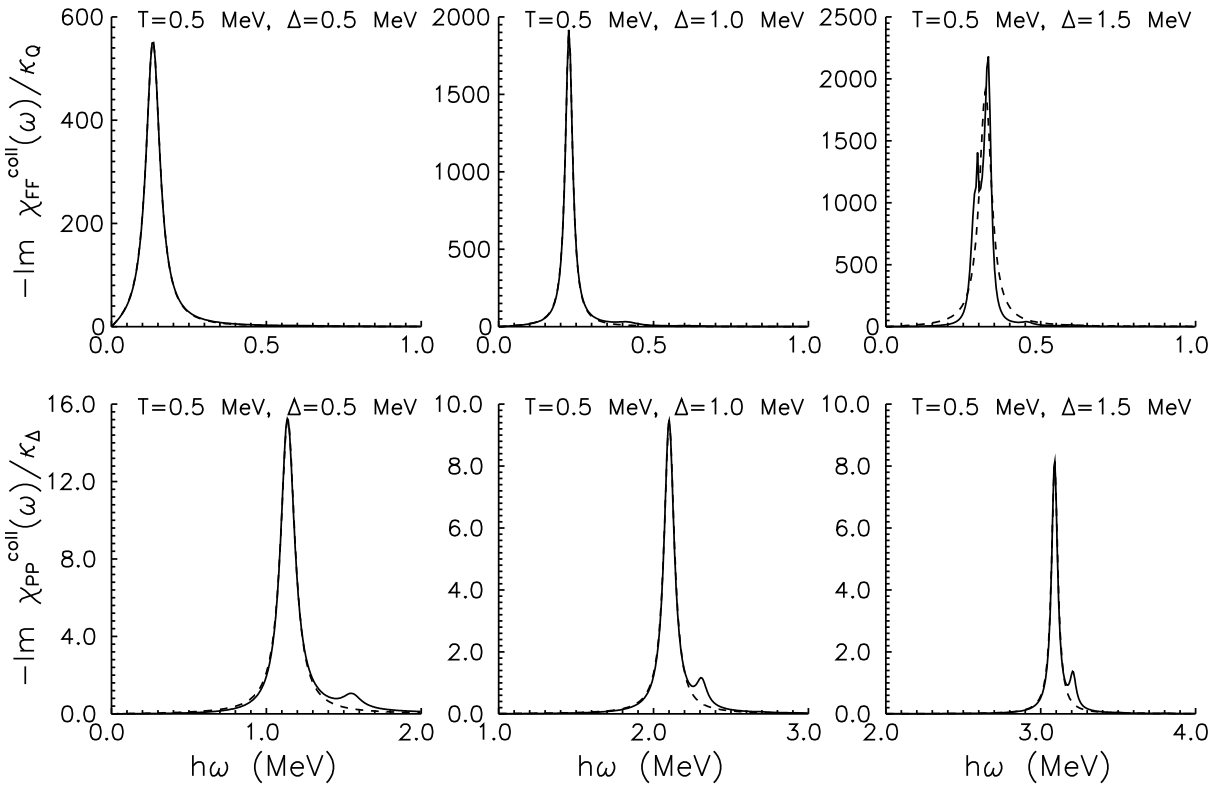

Figure 7: The frequency dependence of the imaginary part of collective FF- and PP-response functions, associated to the intrinsic functions shown in Fig.3. The solid and dashed curves correspond to (102),(104) and to the fit to (102),(104) by the response function of a damped oscillator.

nately, in reality one often encounters a different situation in that the collective strength splits over several closely placed peaks which may even overlap. This situation is typical for a multi-dimensional case, for which it has been demonstrated in [11] of how transport coefficients may still be deduced through a generalized fitting procedure. Indeed, most likely the present situation ought also be treated in this way. Unfortunately, the method suggested in [11] is tedious and very time consuming. In order to circumvent problems of this type we prefer to define transport coefficients through the approximation defined in (21), which still exploits the information contained in the form $(\boldsymbol{\kappa}\boldsymbol{\chi}_{\rm coll}{}^{-1}(\omega)\boldsymbol{\kappa})_{\mu\nu}$ of the collective response tensor. The fit to the polynomial of second order given on the right hand side of (21) can easily be done for any function, even if $\boldsymbol{\chi}_{\rm coll}{}^{-1}(\omega)$ does not look much like a polynomial. In such cases the transport coefficients may be considered as some frequency average over the interval of the fit. Unfortunately, averaging in frequency does not change much the fluctuations with respect to the deformation. That is why both quantities were also averaged over the deformation like before. The results of such a procedure are shown in Fig.8 for the diagonal components of friction and inertia. As it is seen from the figure, the transport coefficients obtained in this way exhibit fluctuations with deformation similar, but noticeably different to those of the zero frequency limit, which have been shown before in Fig.6. Moreover, the mean value of both components of friction and inertia tensor now are larger than before. The difference is especially large in case of $\Delta\Delta$-friction, the reason being that at small ω the imaginary part of the response function for the pairing degree of freedom $\widehat{\chi}_{PP}(\omega)$ increases with ω much slower as com-

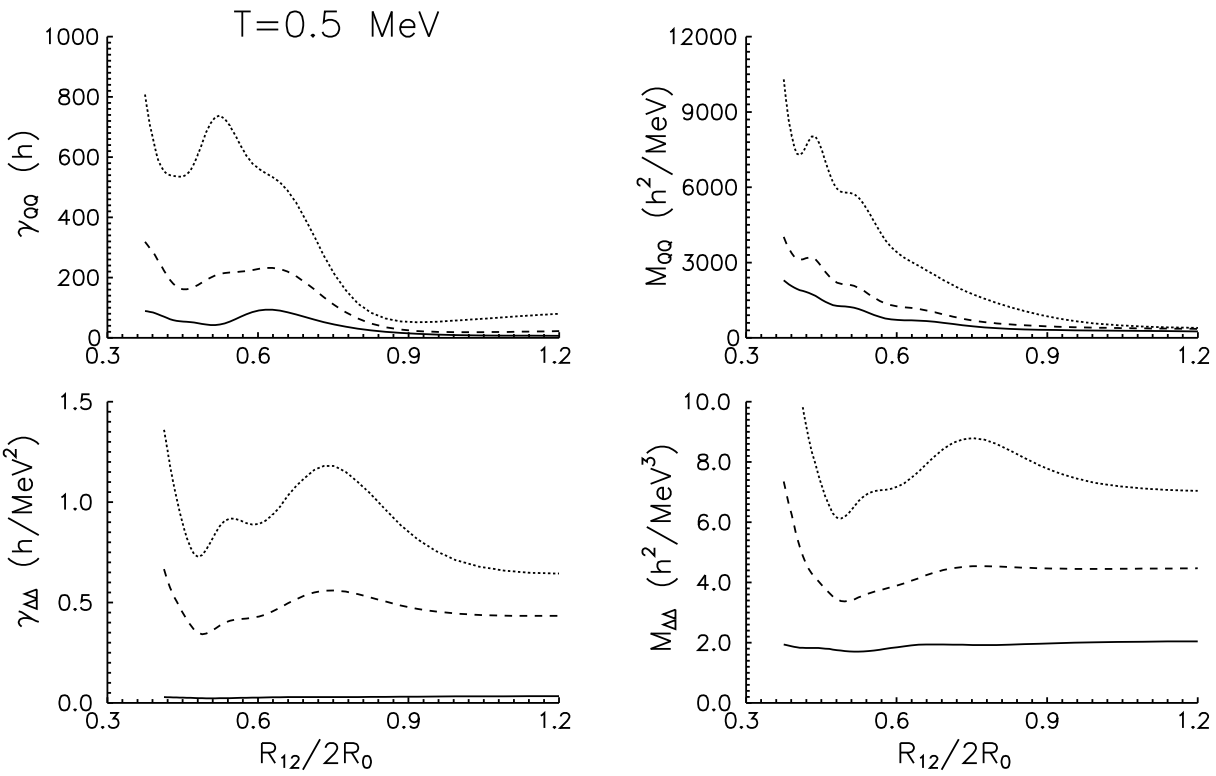

Figure 8: The components of the friction (left-hand-side) and inertia (right-hand-side) tensor obtained by a fit to (102),(104) through the response function of a damped oscillator according to (21) as function of the deformation for temperature $T = 0.5$ MeV. The dotted, dashed and solid curves correspond to $\Delta = 0.5, 1.0$ and 1.5 MeV.

pared with that of a damped oscillator. Consequently, $\gamma_{\Delta\Delta}(0)$ is smaller than $\gamma_{\Delta\Delta}$. This is a clear hint that in such a case the zero frequency limit may not be considered a good approximation.

5.3 Temperature dependence of transport coefficients

In the past various models have been presented to justify nuclear dissipation, even more exist to evaluate the friction coefficient numerically. In [33] a compilation of data has been put together in which theory is confronted with experimental evidence. The latter is obtained by comparing solutions of the "macroscopic equations" (of Fokker-Planck or Langevin type) with experimentally observable quantities, the input into such equations being chosen phenomenologically. It was seen that the predictions which the theoretical models give for the effective, local collective width $\Gamma = \gamma/M$ of the fission mode — sometimes referred to as the "reduced friction coefficient" β— deviate by as much as two orders of magnitude. It can be said that the results of [34] classified there by "linear response theory" (LRT) were in good agreement with those required by experimental evidence. As compared to the application of LRT mentioned in [33], new calculations have in the mean time been presented in [8, 30]. Grossly speaking their results lie in the same regime as those of [34].

As all such microscopic calculations depend on some quantities which are not known all too precisely, it has been argued previously (see e.g.[9, 30]) that one should look for the

temperature dependence of the effective transport coefficients. Indeed, here the various theories differ from one another in very pronounced ways. The wall formula, for instance, predicts a friction coefficient which is practically independent of T. As demonstrated in [32, 9], such a picture is obtained as the macroscopic limit of LRT, provided the latter is applied to purely independent particle motion, viz if all influences of collisions are discarded. Truth is that the latter have important implications on the T-dependence of friction. Two body viscosity, for instance, comes about in the regime of "collisional dominance", in which viscosity is known to decrease with temperature, in ideal cases like $1/T^2$. Although the notion "viscosity" intrinsically refers to a nuclear liquid, in the case of genuine "collisional dominance" such a T-dependence also prevails for the friction coefficient of the low frequency collective modes of finite nuclei, see e.g. [35]. A similar dependence was found in [36] and [37]. It is true that these models do not rely on a hydrodynamic description, but in each one collisions are assumed to dominate collective dynamics in one way or another. In [32] and [9] it has been discussed how this behavior of damping may be obtained within LRT.

Fortunately, a possible T-dependence of nuclear dissipation has also been studied by interpreting the experimental results, see [38, 39, 40]. Here we shall not pursue any details of these studies. It may suffice to summarize the general claims put forward in these publications, which is that dissipation increases with T at small excitations, leveling off or even decreasing at larger temperatures. Indeed, generally speaking, such behaviour was found in our previous calculations [32], at least qualitatively. How large friction may become at intermediate temperature and how quickly it decreases afterwards is still an unsettled question. It depends both on the contributions from the heat pole as well as on the approximations in which collisions are treated, for details see [32] and [9]. As stressed earlier, in the present paper we mainly address the regime of very small temperatures where pairing must not be neglected. Unfortunately, to the best of our knowledge, in this regime, neither experimental information is available, nor to date is there any theoretical model with which our results might be compared. Our predictions are as follows.

The ones for the fission mode are displayed in Fig.9. To simplify matters the pairing mode has been treated in equilibrium, or "local" equilibrium, rather, if the latter is being defined with respect to the collective variable Q. In other words, at given shape the gap equation (57) has been solved for various temperatures. The resulting Δ's then have been used for the calculation of the transport coefficients for the Q-mode displayed in Fig.9. For the system considered pair correlations disappear above $T \simeq 0.5$ MeV. Indeed, it is only below such temperatures that the transport coefficients deviate from the unpaired case. At first sight the deviation may look small. A closer look, however, shows that important modifications of the nature of the transport process are present. Neglecting any quantum corrections (for collective dynamics) one realizes that for smaller temperatures nuclear dissipation may get so weak that one reaches Kramers' so called "low viscosity" limit". This is very interesting on recalling that in nuclear physics experimental results are commonly interpreted on the basis of Kramers' simple formula for the "high viscosity limit". Moreover, it so happens that in this regime quantum effects in collective dynamics are present. They lead to additional modifications of the decay rate. Unfortunately, at present one is able to account for these effects only above some critical temperature, which turns out to be less than the T where pairing disappears. There is no room to elaborate further on details of collective dynamics. Instead we like to refer to a recent Letter [41], as well as to forthcoming papers.

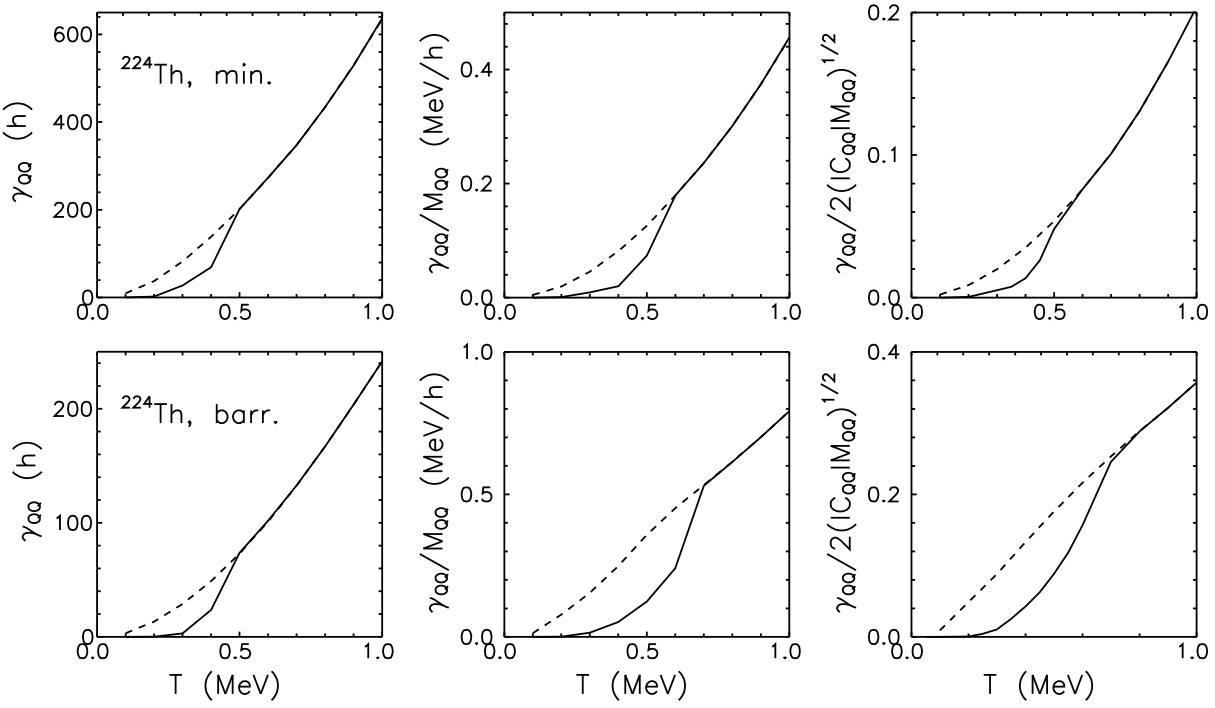

Figure 9: The average QQ-friction coefficient γ_{QQ}, reduced friction coefficient $\beta_{QQ} \equiv \gamma_{QQ}/M_{QQ}$ and the damping factor $\eta_{QQ} = \gamma_{QQ}/2\sqrt{C_{QQ}M_{QQ}}$ at the ground state (top) and fission barrier (bottom) of ^{224}Th as function of temperature. The dashed curves show the results obtained neglecting pairing.

Let us turn to the pairing mode Δ now, for which in Fig.10 we show the damping factor $\eta_{\Delta\Delta} = \gamma_{\Delta\Delta}/2\sqrt{C_{\Delta\Delta}M_{\Delta\Delta}}$ as function of temperature for various values of Δ. Here, the latter is to be understood as a collective variable, not fixed by the gap equation. The values presented in this figure represent averages along the fission valley. The $\eta_{\Delta\Delta}$ is seen to be very small, implying the corresponding pairing vibrations to be strongly *under*-damped. Most likely this means that on the way from saddle to scission the pairing mode does *not equilibrize*. The consequences such an observation may have on odd-even effects in mass distributions etc. will have to be the subject of some further studies.

6 Summary

A detailed study of transport coefficients has presented for temperatures below $T = 1MeV$ which finally aim at a description of the dynamics of fissioning nuclei. Pairing effects were accounted for within the independent quasi-particle approximation. An appropriate treatment of particle number conservation has been achieved by modifying the response functions of the locally harmonic approximation. For harmonic vibrations without damping it has been demonstrated that the same secular equation for the collective frequencies is obtained as those derived earlier within common RPA. In our version, the pairing gap Δ is introduced as an independent dynamical variable, similar to the parameters specifying the shape of nuclear surface. The response functions and the tensors of collective friction and inertia have been calculated for a realistic, deformed Woods-Saxon potential. The components of friction and inertia tensors were examined as function of the pairing gap

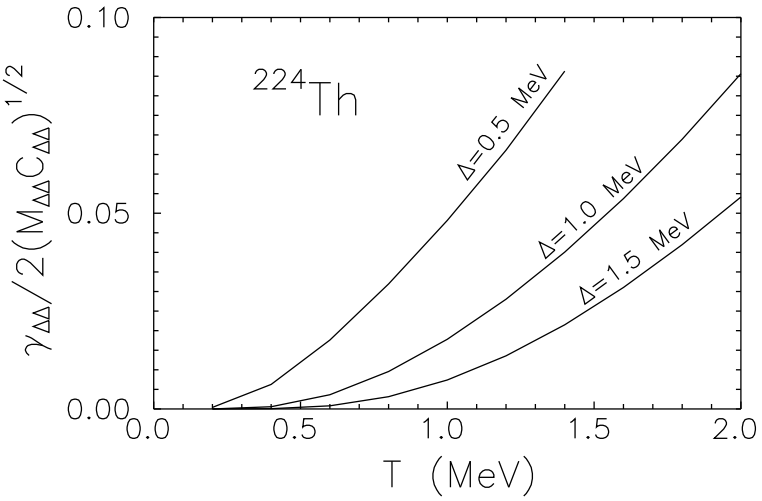

Figure 10: The damping parameter of pairing vibrations $\bar{\eta}_{\Delta\Delta} = \gamma_{\Delta\Delta}/2\sqrt{C_{\Delta\Delta}M_{\Delta\Delta}}$ as the function of temperature for few fixed values of Δ. The damping parameter is averaged along the fission valley of nucleus ^{224}Th.

and the deformation of the nucleus. With respect to the latter strong variations were found of all transport coefficients. They are caused both by shell effects as well as by avoided crossings of single-particle levels. It has been argued that such variations may be considered as carrying unphysical features, for which reason they have been smoothed out by averaging over the shape in such a way as to keep gross shell effects. As could be expected, the friction coefficient decreases with increasing Δ. Finally we may say that the temperature dependence of the friction coefficient obtained in our model is in qualitative agreement with the conclusions reached in [38, 39, 40], namely that dissipation increases with T at small excitations.

Acknowledgments. The authors want to thank the Deutsche Forschungsgemeinschaft for financial support and to D. Einzel, A.S. Jensen, K. Hara, K. Pomorski and V. Abrosimov for fruitful discussions and suggestions. The use of V.V. Pashkevich's computational code for the quantities based on the shell model with deformed Woods-Saxon potential is greatly acknowledged. One of us (F.A.I) would like to thank the Physik Department of the TUM for the hospitality extended to him during his stay in Garching.

A Intrinsic response functions

The time dependent response function $\tilde{\chi}_{FF}(t)$ is defined by (11). To obtain $\tilde{\chi}_{PN}(t)$ one should replace $\hat{F}^I(t), \hat{F}^I(s)$ in (11) by $(\hat{P}+\hat{P}^\dagger)^I(t), \hat{N}^I(s)$. In the same way one can obtain expressions for the other response functions, too. For the operators $\hat{P}$ and $\hat{N}$ one may use the quasiparticle representation like (72)

$$\hat{P}+\hat{P}^\dagger = \sum_k 2u_kv_k - \sum_k 2u_kv_k(\alpha_k^\dagger\alpha_k + \alpha_{\bar{k}}^\dagger\alpha_{\bar{k}}) + \sum_k (u_k^2 - v_k^2)(\alpha_k^\dagger\alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}}\alpha_k)$$
$$\hat{N} = \sum_k 2v_k^2 + \sum_k (u_k^2 - v_k^2)(\alpha_k^\dagger\alpha_k + \alpha_{\bar{k}}^\dagger\alpha_{\bar{k}}) + \sum_k 2u_kv_k(\alpha_k^\dagger\alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}}\alpha_k) \quad (120)$$

The final expressions for FP or FN-response functions turn out to be much simpler than the one presented in (80) since they contain only diagonal sums ($k = j$).

$$\begin{aligned}
\chi_{FN}(\omega) &= \sum_k (2n_k^T - 1)(2u_k v_k)^2 F_{kk} \left(\frac{1}{\hbar\omega - 2E_k + i\varepsilon} - \frac{1}{\hbar\omega + 2E_k + i\varepsilon} \right) \\
\chi_{FP}(\omega) &= \sum_k (2n_k^T - 1) 2u_k v_k (u_k^2 - v_k^2) F_{kk} \left(\frac{1}{\hbar\omega - 2E_k + i\varepsilon} - \frac{1}{\hbar\omega + 2E_k + i\varepsilon} \right) \\
\chi_{NN}(\omega) &= \sum_k (2n_k^T - 1)(2u_k v_k)^2 \left(\frac{1}{\hbar\omega - 2E_k + i\varepsilon} - \frac{1}{\hbar\omega + 2E_k + i\varepsilon} \right) \\
\chi_{NP}(\omega) &= \sum_k (2n_k^T - 1) 2u_k v_k (u_k^2 - v_k^2) \left(\frac{1}{\hbar\omega - 2E_k + i\varepsilon} - \frac{1}{\hbar\omega + 2E_k + i\varepsilon} \right) \\
\chi_{PP}(\omega) &= \sum_k (2n_k^T - 1)(u_k^2 - v_k^2)^2 \left(\frac{1}{\hbar\omega - 2E_k + i\varepsilon} - \frac{1}{\hbar\omega + 2E_k + i\varepsilon} \right)
\end{aligned} \tag{121}$$

The account for the collisional damping for FP- FN- and other response function is carried out in the same way as for FF-response and leads to

$$\begin{aligned}
\chi_{FN}(\omega) &= \sum_k (2u_k v_k)^2 F_{kk} \sum_{s \neq s'} \chi_{ksks'}(\omega) \\
\chi_{FP}(\omega) &= \sum_k 2u_k v_k (u_k^2 - v_k^2) F_{kk} \sum_{s \neq s'} \chi_{ksks'}(\omega) \\
\chi_{NN}(\omega) &= \sum_k (2u_k v_k)^2 \sum_{s \neq s'} \chi_{ksks'}(\omega) \\
\chi_{NP}(\omega) &= \sum_k 2u_k v_k (u_k^2 - v_k^2) \sum_{s \neq s'} \chi_{ksks'}(\omega) \\
\chi_{PP}(\omega) &= \sum_k (u_k^2 - v_k^2)^2 \sum_{s \neq s'} \chi_{ksks'}(\omega)
\end{aligned} \tag{122}$$

with $\chi_{ksks'}(\omega)$ given by (86)

References

[1] A.Bohr and B.Mottelson, Nuclear Structure, vol.2 (Benjamin, New York, 1975)

[2] Ring and P. Schuck, The Nuclear Many Body Problem (Springer, Berlin, 1980)

[3] T.Lederberger and H.C.Pauli, Nucl.Phys. A 207 (1973) 1

[4] L.G.Moretto, Phys. Lett. 40 B (1972) 1; see also L.G.Moretto and R.P.Babinet, Phys. Lett. 49 B (1974) 147; L.G.Moretto, Phys. Lett. 57 B (1975) 317

[5] A.Staszczak, S.Pilat and K.Pomorski, Nucl. Phys. A 504 (1989) 589

[6] A.S. Jensen, J. Leffers, K. Reese and P.J. Siemens Phys. Lett. 117 B (1982) 157

[7] A.S. Jensen, J. Leffers, K. Reese, H. Hofmann and P.J. Siemens, Phys. Lett. 117 B (1982) 5; see also A. S. Jensen, J. Leffers, H. Hofmann and P.J. Siemens, Physica Scripta Vol. T5 (1983) 186

[8] F.A.Ivanyuk, H.Hofmann, V.V.Pashkevich and S.Yamaji, Phys.Rev. C 55 (1997) 1730

[9] H. Hofmann, Phys. Rep. 284 (4&5), (1997) 137

[10] P.J. Siemens and A.S. Jensen, Elements of Nuclei: Many-Body Physics with the Strong Interaction (Addison and Wesley, 1987)

[11] R.Samhammer, H. Hofmann and S.Yamaji, Nucl. Phys. A 503 (1989) 404

[12] D. Kiderlen, H. Hofmann and F.A. Ivanyuk, Nucl. Phys. A 550 (1992) 473

[13] D. Kiderlen, Diploma Thesis, TU Munich (1990)

[14] N.N.Bogolyubov, D.N.Zubarev and Yu.A.Tserkovnikov, Sov.Phys.Dokl. 2 (1957) 535

[15] M.Sano and S.Yamasaki, Prog. of Theor. Phys. 29 no.3 (1963) 397

[16] J.Högaasen-Feldman, Nucl.Phys. 28 (1961) 258

[17] V.I.Abrosimov, V.M.Strutinsky, Sov.Jour.Nucl.Phys. 27 (1978) 1508

[18] V.I.Abrosimov, Sov.Jour.Nucl.Phys. 31 (1980) 348

[19] D.R.Bes, Z.Szymanski, Nuovo Cim. 26 (1962) 787

[20] V.G.Soloviev, Nucl.Phys. 69 (1965) 1

[21] R.Y.Cusson and K.Hara, Zeit. f Phys. 209 (1968) 428

[22] A.S.Jensen, H.Hofmann, P.Siemens, in: S.S.Wu, T.T.S.KUO (Eds.), Nucleon-Nucleon Interaction and the Nuclear Many-Body Problems (World Scientific, Singapore, 1984)

[23] V.M. Strutinsky, Nucl. Phys. A 95 (1967) 420; A 122 (1968) 1

[24] M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky and C. Y. Wong, Rev. Mod. Phys. 44 (1972) 320

[25] M.Brack and P.Quentin, Nucl.Phys. A 361 (1981) 35

[26] G.Baym, C.J.Pethik, Landau Fermi-Liquid Theory, Willey, New York, 1991

[27] P.Bhattacharyya, C.J.Pethik, H.Smith, Phys.Rev. B 15 (1977) 3367

[28] C.J.Pethik, H.Smith, P.Bhattacharyya, Phys.Rev. B 15 (1977) 3384

[29] P.J. Siemens and A.S. Jensen, Single particle dynamics in: eds. R. Broglia, G.B. Hagemann and B. Herskind (Elsevier Science Publishers B.V., Amsterdam, 1986)

[30] S. Yamaji, F.A. Ivanyuk and H. Hofmann, Nucl. Phys. A 612 (1997) 1

[31] V.V. Pashkevich, Nucl. Phys. A 169 (1971) 275

[32] H. Hofmann, F.A. Ivanyuk and S. Yamaji, Nucl. Phys. A 598 (1996) 187

[33] D.Hilscher I.I.Gonchar and H.Rossner, Physics of Atomic Nuclei, 57 (1994) 1187

[34] S. Yamaji, H. Hofmann and R. Samhammer, Nucl. Phys. A475 (1988) 487

[35] A.G.Magner and H.Hofmann, to be published

[36] S. Ayik and W. Nörenberg, Z. Phys. A309 (1982) 121

[37] B.W.Bush, G.F.Bertsh and B.A.Brown, Phys.Rev. C 45 (1992) 1709

[38] D.J.Hofman, B.B.Back, I.Diószegi, C.P.Montoya, S.Schadmand, R.Varma, and P.Paul, Phys.Rev.Let. 72 (1994) 470; P.Paul and M.Thoennessen, Ann.Rev.Part. Nucl.Sci. 44 (1994) 65

[39] J. Wilczyński, K. Siwek-Wilczyńska and H.W. Wilschut, Phys. Rev. C 54 (1996) 325

[40] G. Rudolf, RIKEN-Symposium, March 1998

[41] H. Hofmann and F.A. Ivanyuk, Phys. Rev. Lett. 82 (1999) 4603; nucl-th/9904069